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STELLAR PARALLAXES FROM PHOTOGRAPHS MADE WITH THE 40-INCH REFRACTOR OF THE YERKES OBSERVATORY

By FREDERICK SLOCUM and S. A. MITCHELL

In the *Astrophysical Journal*, Vols. **32**, **33**, and **34**, Professor Schlesinger has published a series of articles on parallax determinations with the 40-inch refractor, and in those articles may be found full descriptions of the apparatus and the methods used in the present investigation.

In but a few respects have we departed from his practice. All of our plates have been taken through a yellow color-filter placed in the plate-holder immediately in front of the photographic plate. For the effects of this procedure see "The Function of a Color-Filter and 'Isochromatic' Plate in Astronomical Photography," by R. J. Wallace, *Astrophysical Journal*, **27**, 106, 1908, and also "On the Errors in Photographic Positions Caused by Observing through Glass," by Frank Schlesinger, *Publications of the Allegheny Observatory*, Vol. **1**, No. 14, p. 101.

Again we have, in general, put two exposures upon a plate instead of three. This has necessitated a revision of the system of weights—and we have adopted the following table of weights (p. 2).

Professor S. A. Mitchell, of Columbia University, as research assistant professor at the Yerkes Observatory for the year 1912–1913, has assisted in making the observations during the past

year and has measured nearly half of the plates used in the present investigation. Mr. Sullivan, the electrician of the Yerkes Observatory, has, as usual, rendered valuable assistance at the telescope. Miss M. M. Hopkins of Smith College, while research assistant during the summer of 1912, measured and reduced one of the fields. Others have from time to time shared in the observational work. The abbreviations in the column marked "Observers" have the following significance: F = Fox, J = Jordan, M = Mitchell, Sl = Slocum, Su = Sullivan, V = Van Maanen.

TABLE OF WEIGHTS

Number of Exposures	Quality of Images		
	Good	Fair	Poor
1	0.7	0.5	0.3
2	1.0	0.7	0.4
3	1.2	0.9	0.5

Miss Eudora Magill, since June 1912 at Yerkes Observatory, has assisted in practically all of the computations.

In the tables and reductions, unless stated to the contrary, all values are in terms of scale divisions. One division of the scale is $\frac{1}{4}$ mm and equals 2″.66.

In eleven of the fields the displacement has been measured parallel to the equator, and in three, parallel to the ecliptic. The solution in each case yields the proper motion for 100 days. Where the measurements have been made parallel to the equator, the proper motion has also been given in seconds of time per year.

ϕ Andromedae (1^h4^m, $+46°43'$)

This is a binary system 0Σ *515*, with components of 4.9 and 6.5 magnitude. The companion has described an arc of more than 90° and, according to the latest observations, was about 0″.25 distant. The proper motion of the system according to Boss is $+0^s.0007$, $-0''.007$, but according to Auwers $-0^s.0034$, $-0''.012$. The sign of the latter value for the μ_a agrees with the results obtained from the plates.

The magnitude of the pair is 4.28, so the rotating disk was used

to reduce the brightness. Eight plates were secured as described in Table 1. They were measured by Mr. Slocum.

TABLE 1

PLATES OF ϕ *Andromedae*

No.	Date	Hour Angle	Observers	Quality of Images	Remarks
14	1907 Aug. 17	-1^h5	J, Su, J	Fair	
19	Aug. 24	−0.2	J, Su, J	Good	
21	Aug. 31	−1.0	J, Su, J	Good	
56	1908 Jan. 4	+0.9	J, J, J	Good	Telescope
166	1909 Dec. 19	−0.9	Sl, Sl, Sl	Fair	east
174	Dec. 23	−0.3	Sl, Sl, Sl	Good	
279	1910 July 30	−1.2	Su, Sl	Good	
292	Aug. 26	−1.8	Su, Su	Fair	

COMPARISON STARS

No.	Diameter	*X* (longitude)	*Y* (latitude)	Dependence
	mm			
2	0.15	−102.5	+345.5	+0.19
4	.17	+376.7	+129.7	+ .11
5	.20	−220.7	+ 51.8	+ .21
6	.40	+256.3	+ 10.2	+ .12
9	.31	−338.9	−156.8	+ .22
10	.15	+ 29.1	−380.4	+0.15
Parallax star	0.24	− 65.26	+ 0.93	.....

The mean magnitude of the comparison stars is 9.2.

TABLE 2

REDUCTIONS FOR ϕ *Andromedae*

Plate	Solution (m)	Weight (p)	Parallax Factor (P)	Time in Days (t)	Residual (v)	$\sqrt{p} \cdot v$ in Arc
14	−1.410	0.9	+0.956	−507	+0.008	+0.″02
19	1.417	1.2	+0.909	−500	+ .001	+ .00
21	1.427	1.2	+0.850	−493	− .009	− .03
56	−1.422	1.2	−0.913	−367	+ .002	+ .01
166	1.431	0.9	−0.778	+348	+ .014	+ .04
174	1.460	1.2	−0.819	+352	− .014	− .04
279	−1.450	1.0	+1.014	+571	.000	.00
292	−1.449	0.7	+0.892	+598	+0.002	0.00

The normal equations are:

$$\begin{aligned} 8.3c - \ 3.6310\mu + 1.8310\pi &= -11.8924 \\ +181.8672\mu - 7.1963\pi &= +\ 4.6465 \\ +6.6162\pi &= -\ 2.5946 \end{aligned}$$

from which

$$\begin{aligned} c &= -1.4345 \\ \mu &= -0.0030 = -0''.008 \\ \pi &= +0.0015 = +0''.004 \pm 0''.008 \end{aligned}$$

Probable error corresponding to unit weight, $\pm 0.0072 = \pm 0''.019$.

48 Cassiopeiae (1^h54^m, $+70°25'$)

This star is a visual binary, discovered by Burnham in 1878. The components are 5.0 and 7.5 magnitude. At the time of discovery the distance was a trifle over $1''.0$, but since then it has decreased to approximately $0''.3$ and the position angle has changed about 200°. Ten plates were secured as described in Table 1. They were measured by Mr. Slocum. The rotating disk was used to reduce the brightness of the parallax star.

TABLE 1

PLATES OF *48 Cassiopeiae*

No.	Date			Hour Angle	Observers	Quality of Images
57	1908	Jan.	4	$+1^h0$	J, Su, J	Fair
102	1909	Sept.	4	−0.8	Sl, Su, Sl	Fair
136		Oct.	30	− .9	Su, Sl, Su	Fair
175		Dec.	23	− .2	Sl, Su, Sl	Fair
196	1910	Jan.	9	− .4	Sl, Sl	Fair
286		Aug.	20	− .7	Su, Sl	Fair
470	1911	Aug.	19	−1.4	Su, V	Good
591		Dec.	18	− .4	V, Su	Good
611		Dec.	25	− .6	V, Su	Good
638	1912	Jan.	20	−0.1	Sl, V	Good

COMPARISON STARS

No.	Diameter	*X* (longitude)	*Y* (latitude)	Dependence
	mm			
1	0.21	−171.0	− 72.5	0.155
2	.11	−167.6	+147.8	.13
3	.13	− 77.5	−315.9	.19
4	.15	+100.9	+272.9	.15
5	.14	+107.4	− 78.2	.19
6	.17	+207.8	+ 45.9	0.185
Parallax star	0.29	+ 10.62	− 16.82	.....

The mean magnitude of the comparison stars is approximately 10.5.

TABLE 2

REDUCTIONS FOR *48 Cassiopeiae*

Plate	Solution (m)	Weight (p)	Parallax Factor (P)	Time in Days (t)	Residual (v)	$\sqrt{p}\cdot v$ in Arc
57	−0.273	0.9	−0.667	−719	+0.023	+0.″06
102	.338	.9	+0.988	−110	− .019	− .05
136	.326	.9	+0.396	− 54	− .005	− .01
175	.341	.9	−0.506	− 0	− .018	− .05
196	.320	.7	−0.734	+ 17	+ .003	+ .01
286	.350	.7	+1.010	+240	− .018	− .04
470	.317	1.0	+1.009	+604	+ .029	+ .08
591	.340	1.0	−0.421	+725	+ .010	+ .03
611	.339	1.0	−0.528	+732	+ .011	+ .03
638	−0.372	1.0	−0.843	+758	−0.022	−0.06

The normal equations are:

$$9.0c + 22.0420\mu - 0.3999\pi = -2.9872$$
$$+252.0131\mu - 2.4578\pi = -8.0474$$
$$+4.9265\pi = +0.1353$$

from which

$$c = -0.3229$$
$$\mu = -0.0037 = -0.''010$$
$$\pi = -0.0006 = -0.''002 \pm 0.''016$$

Probable error corresponding to unit weight, $\pm 0.0137 = \pm 0.''036$

20 Persei (2^h47^m, $+37°56'$)

This star, β *324*, consists of two stars of magnitudes 6.0 and 6.7 whose distance never much exceeds 0".2. According to Burnham, an approximate orbit has been found giving a period of 27.7 years, though "these results are of somewhat doubtful value from uncertainty in the adjustment of the measures as to quadrants." 14" distant from this double *AB* is a star *C* of 10th magnitude which must also be a member of the system. The star is of spectral type F. Thirteen plates of this star were obtained. They were measured by Mr. Mitchell. The reductions were carried out at Columbia University by Miss Magill.

TABLE 1

PLATES OF *20 Persei*

No.	Date	Hour Angle	Observers	Quality of Images
20	1907 Aug. 24	-1^h2	J, Su, J	Poor
23	Aug. 31	−1.0	Su, J, Su	Fair
78	1908 Nov. 28	−0.4	F, Su, F	Good
182	1910 Jan. 6	−0.9	Sl, Sl, Sl	Fair
197	Jan. 9	−0.5	Sl, Sl	Good
287	Aug. 20	−0.8	Su, Sl	Fair
295	Aug. 26	−0.9	Su, Su	Fair
302	Sept. 9	−1.1	Su, Su	Good
362	1911 Jan. 21	+0.1	Sl	Poor
472	Aug. 19	−1.0	Sl, Su	Fair
487	Sept. 2	−1.6	Su, Sl	Good
629	1912 Jan. 8	−0.8	V, Sl	Good
649	Feb. 4	+0.1	Sl, Sl	Good

COMPARISON STARS

No.	Diameter	*X* (right ascension)	*Y* (declination)	Dependence
	mm			
4	0.22	−174	+235	+0.222
6	.14	−124	−180	.235
9	.19	+ 3	−144	.208
10	.15	+ 83	+278	.167
11	.22	+212	−189	+0.168
Parallax star	0.22	− 17.8	− 5.3	.....

The average magnitude of the comparison stars is 9.3. The rotating disk was used to cut down the light of the parallax star.

TABLE 2

REDUCTIONS FOR *20 Persei*

Plate	Solution (m)	Weight (p)	Parallax Factor (P)	Time in Days (t)	Residual (v)	$\sqrt{p}\cdot v$ in Arc
20.....	+0.008	0.5	+0.923	−738	−0.013	−0″.02
23.....	.014	0.9	+ .885	−731	− .008	− .02
78.....	.071	1.2	− .355	−276	+ .015	+ .04
182.....	.092	0.9	− .825	+128	+ .008	+ .02
197.....	.072	1.0	− .848	+131	− .012	− .03
287.....	.094	0.7	+ .942	+354	+ .004	+ .01
295.....	.083	0.7	+ .935	+360	− .008	− .02
302.....	.108	1.0	+ .809	+374	+ .016	+ .04
362.....	.092	0.3	− .914	+508	− .016	− .02
472.....	.109	0.7	+ .946	+718	− .004	− .01
487.....	.113	1.0	+ .872	+732	− .001	.00
629.....	.127	1.0	− .837	+862	− .003	− .01
649.....	+0.130	1.0	−0.872	+889	−0.002	0.00

The normal equations are:

$$10.9c + 28.999\mu + 0.9154\pi = +0.9624$$
$$370.2357\mu - 7.6200\pi = +4.4552$$
$$7.5998\pi = -0.0153$$

These yield

$$c = +0.0719$$
$$\mu = +0.0063 = +0''.017 \qquad = +0''.005 \text{ per year}$$
$$\pi = -0.0043 = -0''.012 \pm 0''.007$$

Probable error corresponding to unit weight $= \pm 0.0068 = \pm 0''.018$.

No other determination of this parallax has been published.

9 Camelopardalis (4^h44^m, $+66°10'$)

This is a 4.4-magnitude star with a B-type spectrum. Frost and Adams (*Astrophysical Journal*, **19**, 350, 1904) find it to be a spectroscopic binary with very sharp H and K lines which yield variable radial velocity different from that given by the broad lines. The system has been further investigated by O. J. Lee at the Yerkes Observatory (*Astrophysical Journal*, **37**, 1, 1913). Ten plates of this star were obtained for parallax determinations. They were measured by Mr. Mitchell.

TABLE 1

PLATES OF 9 *Camelopardalis*

No.	Date	Hour Angle	Observers	Quality of Images
36	1907 Oct. 5	−1h0	J, J	Poor
129	1909 Oct. 16	−0.6	Su, Sl, Su	One good
138	Oct. 30	0.0	Su, Sl, Su	Fair
199	1910 Jan. 9	−1.1	Su, Sl	Good
364	1911 Jan. 21	−0.8	Su, Sl	Fair
491	Sept. 2	−1.6	Sl, Su	Fair
626	1912 Jan. 1	0.0	Su, Sl	Fair
633	Jan. 8	−0.4	Su, Sl	Fair
643	Jan. 20	0.0	Su, Sl	Good
659	Feb. 9	−0.1	Su, Sl	Good

COMPARISON STARS

No.	Diameter	*X* (right ascension)	*Y* (declination)	Dependence
	mm			
2	0.13	−193	+ 13	+0.287
3	.14	−124	−115	.272
4	.30	−128	+120	.243
12	.13	+231	+213	.123
13	.30	+214	+195	+0.076
Parallax star	0.17	− 75.5	− 9.7	.....

The average magnitude of the comparison stars is 8.9. The sector was used to cut down the brilliancy of the parallax star.

TABLE 2

REDUCTIONS FOR 9 *Camelopardalis*

Plate	Solution (*m*)	Weight (*p*)	Parallax Factor (*P*)	Time in Days (*t*)	Residual (*v*)	$\sqrt{p} \cdot v$ in Arc
36	−0.096	0.6	+0.864	−794	+0.008	+0".02
129	.124	0.7	+ .750	− 52	− .019	− .04
138	.104	0.7	+ .570	− 38	− .001	.00
199	.081	1.0	− .577	+ 33	+ .008	+ .02
364	.098	0.7	− .728	+410	− .010	− .02
491	.100	0.7	+ .999	+634	+ .010	+ .02
626	.096	0.7	− .452	+755	− .003	− .01
633	.080	0.7	− .556	+762	+ .011	+ .03
643	.094	1.0	− .714	+774	− .004	− .01
659	−0.087	1.0	−0.902	+794	0.000	0.00

The normal equations are:

$$\begin{aligned} 7.8c + 28.543\,\mu - 1.2665\pi &= -0.7410 \\ 281.6274\mu - 20.4297\pi &= -2.5741 \\ 4.1544\pi &= +0.0775 \end{aligned}$$

These yield

$$\begin{aligned} c &= -0.0959 \\ \mu &= -0.0003 = -0''.001 \qquad = -0''.001 \text{ per year} \\ \pi &= -0.0120 = \mathbf{-0''.032 \pm 0''.011} \end{aligned}$$

Probable error corresponding to unit weight $= \pm 0.0067 = \pm 0''.018$.

μ Orionis (5^h57^m, $+9°39'$)

This star is interesting principally on account of its radial velocity. It is of spectral type A2, and according to Frost (*Astrophysical Journal*, **26**, 264, 1906) is a spectroscopic binary with a period of 0.77 day, the shortest period of any A-type star in Campbell's *Second Catalogue of Spectroscopic Binary Stars*. That this star is not a true member of the *Orion* group is shown by the fact that it has not an *Orion*-type spectrum, and also by the comparatively large radial velocity of +50 km per sec.

μ Orionis is β 1056 with a 14th-magnitude companion in position angle 272°.0 and distance 16″.8. According to Burnham, it is "probably only an optical double." Eight plates were obtained as described in Table 1. These plates were measured by Mr. Mitchell and the reductions were made at Columbia University by Miss Magill.

TABLE 1

PLATES OF *μ Orionis*

No.	Date			Hour Angle	Observers	Quality of Images
46	1907	Oct.	12	+0^h.6	Su, J, Su	Fair
85	1909	Jan.	17	+0.5	F, Su, F	Two fair
130		Oct.	16	−0.8	Su, Sl, Su	Good
140		Oct.	30	+0.1	Su, Sl, Su	Fair
219	1910	Feb.	27	+0.7	Su, Sl	Good
311		Oct.	1	−0.8	Sl, Sl	One fair
373	1911	Feb.	4	+1.5	Su, Sl	Poor
382		Mar.	4	+1.1	Su, Sl	Fair

On Plate 85, the aperture through mistake was 32 inches. Two images only were measured on this plate and one on Plate 311.

COMPARISON STARS

No.	Diameter	X (right ascension)	Y (declination)	Dependence
	mm			
8.....	0.23	−148	+329	+0.173
9.....	.27	−121	−102	.196
10.....	.22	− 70	−203	.205
15.....	.21	+134	−161	.218
16.....	.19	+205	+137	+0.208
Parallax star..	0.13	+ 8.3	− 11.3	.....

The average magnitude of the comparison stars is 9.3. The rotating disk was used to cut down the brightness of the parallax star.

TABLE 2

REDUCTIONS FOR μ *Orionis*

Plate	Solution (m)	Weight (p)	Parallax Factor (P)	Time in Days (l)	Residual (v)	$p \cdot v$ in Arc
46.....	+0.114	0.9	+0.940	−617	−0.007	−0.″02
85.....	.118	0.7	− .463	−154	+0.020	+ .04
130.....	.126	1.2	+ .908	+118	+ .013	+ .04
140.....	.085	0.9	+ .783	+132	− .026	− .07
219.....	.086	1.0	− .926	+252	− .001	.00
311.....	.129	0.5	+ .989	+468	+ .018	+ .03
373.....	.090	0.4	− .707	+594	+ .003	+ .01
382.....	+0.071	0.7	− .950	+622	− .012	−0.03

The normal equations are:

$$6.3c + 7.563\,\mu + 0.9348\pi = +0.6491$$
$$97.6581\mu - 8.3532\pi = +0.5607$$
$$4.6687\pi = +0.1667$$

These yield

$$c = +0.1023$$
$$\mu = -0.0010 = -0.″003 \qquad = -0.^{s}001 \text{ per year}$$
$$\pi = +0.0134 = \mathbf{+0.″036 \pm 0.″016}$$

Probable error corresponding to unit weight $= \pm 0.0115 = \pm 0.″031$

Groningen Area VII, No. 20 ($16^{h}21^{m}, +48°35'$)

This is a 10.7-magnitude star with a proper motion of 1.″22 per year. It is given as No. 20 in *Groningen Publications*, 19, pp. 100 and 114, and as No. 31 in *Groningen Publications*, 20, p. 131. Ten plates were secured as shown in Table 1. They were measured by Mr. Slocum.

TABLE 1

PLATES OF *Groningen Area VII, No. 20*

No.	Date	Hour Angle	Observers	Quality of Images	Remarks
395	1911 Apr. 1	$-0^h.5$	Su	Good	
396	Apr. 8	−1.2	Sl	Good	
423	June 10	−0.3	Su	Good	
439	July 8	− .1	Su	Good	
700	1912 Mar. 17	− .2	Su, V	Good	One exposure
713	Mar. 21	− .8	Su, V	Good	One exposure
725	Apr. 2	− .4	Su, V	Good	One exposure
807	July 7	− .7	Su, Sl	Good	One exposure
812	July 13	− .4	M	Good	
818	July 14	−0.2	Su, Sl	Good	One exposure

COMPARISON STARS

No.	Diameter	*X* (right ascension)	*Y* (declination)	Dependence
	mm			
1	0.15	+308.0	+119.3	+0.18
2	.39	+278.0	−172.1	.16
3	.13	+221.8	+227.2	.18
4	.18	−138.2	+205.8	.175
5	.24	−329.6	−253.7	.15
6	.24	−340.0	−126.5	+0.155
Parallax star	0.26	+ 12.96	+ 12.90	.....

The average magnitude of the comparison stars is approximately 9.5.

TABLE 2

REDUCTIONS FOR *Groningen Area VII, No. 20*

Plate	Solution (m)	Weight (p)	Parallax Factor (P)	Time in Days (t)	Residual (v)	$\sqrt{p}\cdot v$ in Arc
395	−1.003	0.7	+0.814	−265	−0.008	−0".02
396	−0.993	.7	+ .774	−258	− .003	− .01
423	− .966	.7	− .207	−195	− .008	− .02
439	− .925	.7	− .625	−167	+ .017	+ .04
700	− .541	.7	+ .921	+ 86	+ .020	+ .04
713	− .556	.7	+ .897	+ 90	+ .001	.00
725	− .556	.7	+ .797	+102	− .010	− .02
807	− .513	.7	− .622	+198	− .017	− .04
812	− .477	.7	− .697	+204	+ .015	+ .03
818	−0.501	0.7	−0.709	+205	−0.10	−0.02

The normal equations are:

$$\begin{aligned} 7.0c + 0.0000\mu + 0.9191\pi &= -4.9217 \\ +24.6014\mu - 3.0267\pi &= +2.8698 \\ +3.7191\pi &= -0.8474 \end{aligned}$$

from which

$$\begin{aligned} c &= -0.7093 \\ \mu &= +0.1224 = +0''.326 \quad = +0^s.120 \text{ per year} \\ \pi &= +0.0471 = \mathbf{+0''.125 \pm 0''.012} \end{aligned}$$

Probable error corresponding to unit weight, $\pm 0.0084 = \pm 0''.022$. Kapteyn and DeSitter find $\pi = +0''.13$.

Anonymous ($17^h33^m51^s.2$, $+18°36'32''$ [1910.0])

While making the series of plates for the determination of the parallax of *B.D. 18°.3424*, this star attracted our attention both on account of its reddish color, and because of the fact that it does not appear in either the *B.D.* or *A.G.*, while fainter stars in the immediate vicinity are given in both catalogues. Its visual magnitude is 9.1.

At our request Professor Pickering took up the question of the variability of the star, and an examination of 16 plates, exposed between September 20, 1891, and September 1, 1910, showed no signs of change in brightness.

The series of plates for *B.D. 18°.3424* was measured by Miss M. M. Hopkins of Smith College. At first she included the suspicious star among the comparison stars, but it appeared to have a large proper motion and was therefore rejected. The subsequent measurement of four plates extending over 2.4 years gives for the proper motion of the star

$$\mu_\alpha = +0^s.065, \quad \mu_\delta = +1''.00$$

Its position for 1910.0 is $17^h33^m51^s.22$, $+18°36'31''.7$.

Although the star is some distance (13′) from the center of the plates, there are a number of stars around it suitable for comparison

TABLE 1

PLATES OF *Anon.* ($17^h33^m51^s.2$, $+18°36'32''$ [1910.0])

No.	Date	Hour Angle	Observers	Quality of Images
499	1911 July 29	$0^h.0$	Su, Sl	Good
727	1912 Apr. 2	− .5	Sl, V	Good
730	1912 Apr. 7	− .3	V, Sl	Good
850	1912 Aug. 11	−0.2	Sl, Su	Good

COMPARISON STARS

No.	Diameter	*X* (right ascension)	*Y* (declination)	Dependence
	mm			
19.....	0.15	−195.1	+276.0	+0.22
11.....	.20	−153.9	− 77.4	+ .21
12.....	.19	−121.1	−261.0	+ .20
8.....	.17	+139.2	−134.3	+ .13
17.....	.16	+145.1	− 22.0	+ .13
3.....	.12	+185.8	+218.7	+0.11
Parallax star..	0.26	− 41.95	− 5.42	.....

purposes, so a preliminary value of its parallax has been determined from four plates described in Table 1. They were measured by Mr. Slocum.

The mean magnitude of the comparison stars is approximately 9.8.

TABLE 2

REDUCTIONS FOR *Anon.* ($17^h\,33^m\,51^s.2$, $+18°\,36'\,32''$ [1910.0])

Plate	Solution (m)	Weight (p)	Parallax Factor (P)	Time in Days (t)	Residual (v)	$\sqrt{p}\cdot v$ in Arc
449.....	−0.196	1	−0.676	−248	−0.001	0″.00
727.....	+ .115	1	+ .943	000	+ .007	+ .02
730.....	+ .104	1	+ .913	+ 5	− .008	− .02
850.....	+0.162	1	−0.829	+131	0.000	0.00

The normal equations are:

$$4.0c - 1.120\mu + 0.351\pi = +0.1850$$
$$7.869\mu + 0.636\pi = +0.7035$$
$$+2.867\pi = +0.2016$$

from which

$$c = +0.0696$$
$$\mu = +0.0960 = +0''.255 \qquad = +0^s.065 \text{ per year}$$
$$\pi = +0.0405 = +\mathbf{0''.108 \pm 0''.011}$$

Probable error corresponding to unit weight, $\pm 0.0072 = \pm 0''.019$.

B.D. 18°3423 (17^h34^m, $+18°37'$)

This is a 9.0-magnitude star which appears on the plates taken to determine the parallax of *B.D. 18°.3424*. This star was at first included among the comparison stars but was found to have an appreciable proper motion, and was therefore rejected in the investigation of *B.D. 18°.3424*. From the plates, $\mu_\alpha = -0^s.013$, $\mu_\delta = -0''.153$. From two observations given in the *Berlin A.G.*,

p. 129, the corresponding values are $\mu_\alpha = -0^s.012$, $\mu_\delta = -0''.13$. Nine plates were used as shown in Table 1. They were measured by Mr. Slocum.

TABLE 1

PLATES OF *B.D. 18°.3423*

No.	Date	Hour Angle	Observers	Quality of Images
231	1910 Mar. 19	−0h6	Su, Sl	Fair
270	July 30	− .2	Su, Sl	Poor
408	1911 Apr. 22	−1.2	Sl, Su	Good
449	July 29	.0	Su, Sl	Good
727	1912 Apr. 2	− .5	Sl, V	Good
730	Apr. 7	− .3	V, Sl	Good
822	July 21	.− .3	Su, Sl	Good
850	Aug. 11	− .2	Sl, Su	Good
857	Aug. 17	+0.6	Su, M	Fair

COMPARISON STARS

No.	Diameter	X (right ascension)	Y (declination)	Dependence
	mm			
11	0.20	−287.3	− 46.2	+0.24
13	.16	− 0.5	−173.7	.17
17	.16	+ 11.9	+ 9.2	.20
3	.12	+ 52.7	+250.0	.24
16	.24	+223.2	− 39.3	+0.15
Parallax star	0.24	− 19.82	+ 16.61	.....

The mean magnitude of the comparison stars is approximately 9.7.

TABLE 2

REDUCTIONS FOR *B.D. 18°.3423*

Plate	Solution (m)	Weight (p)	Parallax Factor (P)	Time in Days (t)	Residual (v)	$\sqrt{p}\cdot v$ in Arc
231	+0.826	0.7	+0.992	−497	+0.002	0".00
270	.799	0.4	− .690	−364	+ .004	+ .01
408	.743	1.0	+ .792	− 98	+ .002	00
449	.712	1.0	− .675	0	− .007	− .02
727	.673	1.0	+ .944	+248	+ .004	+ .01
730	.662	1.0	+ .914	+253	− .006	− .02
822	.650	1.0	− .580	+358	+ .005	+ .01
850	.640	1.0	− .828	+379	0	.00
857	+0.640	0.7	− .882	+385	+0.001	0.00

The normal equations are:

$$7.8c+9.1600\mu+0.3680\pi=+5.4258$$
$$+73.6584\mu-6.1607\pi=+5.0562$$
$$+5.2552\pi=+0.3992$$

from which

$$c=+0.7200$$
$$\mu=-0.0208=-0''.055 \quad =-0^s.014 \text{ per year}$$
$$\pi=+0.0012=+0''.003\pm0''.004$$

Probable error corresponding to unit weight $\pm0.0033=\pm0''.009$.

B.D. 18°3424 (17^h34^m, $+18°37'$)

This is a star of magnitude 9.2 which, according to the *Berlin A.G.*, should have a proper motion of 1".19 per year. The plates, however, give a much smaller value, viz., $\mu_\alpha=+0^s.004$, $\mu_\delta=-0''.075$, or $\mu_s=0''.092$ in the direction 144°.75.

Twelve plates were secured as shown in Table 1. They were measured by Miss M. M. Hopkins of Smith College, who was a

TABLE 1

PLATES OF *B.D. 18°3424*

No.	Date	Hour Angle	Observers	Quality of Images
231	1910 Mar. 19	-0^h6	Su, Sl	Good
244	1910 Apr. 9	− .7	Su, Sl	Fair
256	June 18	− .6	Su, Sl, H	Good
270	July 30	− .2	Su, Sl	Good
408	1911 Apr. 22	−1.2	Sl, Su	Good
443	July 8	−0.7	Su, Sl	Good
449	July 29	.0	Su, Sl	Fair
727	1912 Apr. 2	− .5	Sl, V	Fair
730	Apr. 7	− .3	V, Sl	Good
822	July 21	− .4	Su, Sl	Good
850	Aug. 11	− .2	Sl, Su	Good
857	Aug. 17	+0.2	Su, M	Fair

COMPARISON STARS

No.	Diameter	*X* (right ascension)	*Y* (declination)	Dependence
	mm			
8	0.17	−214.3	+ 92.3	+0.41
3	.12	−167.8	−261.0	.36
4	.15	+132.2	+170.1	.16
5	.20	+249.9	− 1.4	+.007
Parallax star	0.25	−108.28	− 29.20	.....

volunteer research assistant at this observatory during the summer of 1912.

The mean magnitude of the comparison stars is approximately 9.7. Of the comparison stars originally selected two were found to have appreciable proper motions. These have been investigated separately. See the reduction of *B.D. 18°.3423* and *Anon.* (17^h33^m, $+18°36'$).

TABLE 2

REDUCTIONS FOR *B.D. 18°.3424*

Plate	Solution (m)	Weight (p)	Parallax Factor (P)	Time in Days (t)	Residual (v)	$\sqrt{p}\cdot v$ in Arc
231.....	+1.286	1.0	+0.992	−476	−0.007	−0".02
244.....	1.305	0.7	+ .905	−455	+ .012	+ .03
256.....	1.284	1.2	− .050	−385	− .011	− .03
270.....	1.298	1.0	− .689	−343	+ .002	+ .01
408.....	1.306	1.0	+ .793	− 77	− .003	− .01
443.....	1.326	1.0	− .375	0	+ .015	+ .04
449.....	1.326	0.7	− .674	+ 21	+ .015	+ .03
727.....	1.324	0.7	+ .944	+269	+ .001	.00
730.....	1.326	1.0	+ .914	+274	+ .003	+ .01
822.....	1.328	1.0	− .578	+379	+ .002	+ .01
850.....	1.324	1.0	− .827	+400	− .002	− .01
857.....	+1.311	0.7	−0.881	+406	−0.016	−0.04

The normal equations are:

$$11.0c - 1.3630\mu + 0.3758\pi = +14.4210$$
$$+121.8005\mu - 9.4403\pi = - 1.2914$$
$$+6.1431\pi = + 0.4597$$

from which

$$c = +1.3115$$
$$\mu = +0.0041 = +0".011 \qquad = +0".003 \text{ per year}$$
$$\pi = +0.0010 = \mathbf{+0".003 \pm 0".008}$$

Probable error corresponding to unit weight, $\pm 0.0067 = \pm 0".018$.

96 Herculis (17^h58^m, $+20°50'$)

This star is of magnitude 5.5 and is spectroscopically very interesting. It is of B type, and shows three separate components. Its spectrum has been investigated by Mitchell at the Yerkes Observatory. Preliminary measures were announced at the Ottawa meeting of the Astronomical and Astrophysical Society of America. For parallax determinations twelve plates were secured. They were measured by Mr. Mitchell.

TABLE 1

PLATES OF *96 Herculis*

No.	Date	Hour Angle	Observers	Quality of Images
271	1910 July 30	0h0	Su, Sl	Poor
398	1911 Apr. 8	−1.4	Su, Sl	Fair
402	Apr. 15	−1.2	Su, Sl	Poor
409	Apr. 22	−1.1	Su, Sl	Good
467	Aug. 19	+0.4	Su, Sl	Fair
728	1912 Apr. 2	−0.3	Sl, V	Good
729	Apr. 2	−0.1	Sl, V	Fair
731	Apr. 7	−0.2	V, Sl	Good
823	July 21	−0.1	Su, Sl	Good
851	Aug. 11	+0.1	Su, Sl	Good
867	Aug. 25	−0.1	Sl, Su	Good
872	Aug. 31	+0.4	Su, M	Good

COMPARISON STARS

No.	Diameter	*X* (right ascension)	*Y* (declination)	Dependence
	mm			
1	0.10	−178	+135	+0.300
2	.14	− 14	−255	.348
3	.11	+ 25	+232	.154
4	.25	+167	−112	+0.198
Parallax star	0.14	− 21.4	− 34.7	.....

The average magnitude of the comparison stars is 9.1. The rotating disk was used to cut down the brightness of the parallax star.

TABLE 2

REDUCTIONS FOR *96 Herculis*

Plate	Solution (*m*)	Weight (*p*)	Parallax Factor (*P*)	Time in Days (*t*)	Residual (*v*)	$\sqrt{p}\cdot v$ in Arc
271	+0.018	0.5	−0.616	−617	−0.020	−0.″04
398	.020	0.5	+ .948	−365	− .016	− .03
402	.050	0.4	+ .905	−358	+ .013	+ .02
409	.052	1.0	+ .849	−351	+ .015	+ .04
467	.052	0.7	− .842	−232	+ .012	+ .03
728	.032	1.0	+ .972	− 5	− .006	− .02
729	.025	0.7	+ .972	− 5	− .013	− .03
731	.046	1.0	+ .950	0	+ .008	+ .02
823	.033	1.0	− .497	+105	− .008	− .02
851	.050	1.0	− .769	+126	+ .008	+ .02
867	.048	0.9	− .898	+140	+ .006	+ .02
872	+0.035	1.0	−0.939	+146	−0.008	−0.02

The normal equations are:

$$9.7c - 6.5283\mu + 0.3754\pi = +0.3854$$
$$53.4934\mu - 6.8135\pi = -0.2258$$
$$7.1429\pi = +0.0001$$

These yield

$$c = +0.0401$$
$$\mu = +0.0005 = +0''.001 \qquad = +0^{s}.000 \text{ per year}$$
$$\pi = -0.0016 = \mathbf{-0''.004 \pm 0''.008}$$

Probable error corresponding to unit weight $= \pm 0.0078 = \pm 0''.021$.

17 Lyrae C ($19^{h}4^{m}$, $+32°21'$)

This is a faint star with large proper motion, $1''.752$ per year, discovered by Burnham near *17 Lyrae* ($19^{h}4^{m}$, $+32°21'$). See *M.N.*, **68**, 517, and *M.N.*, **71**, 208. The visual magnitude of *C* is 11.3, the photographic magnitude 12.5. Nine plates were obtained at six parallactic epochs as shown in Table 1. They were measured by Mr. Slocum.

TABLE 1

PLATES OF *17 Lyrae C*

No.	Date	Hour Angle	Observers	Quality of Images
10A	1907 June 11	$-2^{h}0$	F	Good
77	1908 Nov. 28	+3.6	F, A	Good
90	1909 Mar. 20	−1.7	F	Good
105	Sept. 11	0.0	Sl	Fair
245	1910 Apr. 9	−1.4	Su	Good
252	May 14	−0.3	Su	Good
280	Aug. 6	−0.2	Su	Good
282	Aug. 20	−0.4	Sl, Su	Good
288	Aug. 26	−0.4	Su, Su	Fair

COMPARISON STARS

No.	Diameter	*X* (right ascension)	*Y* (declination)	Dependence
	mm			
1	0.23	−188.2	+ 89.9	+0.15
3	.36	− 94.5	− 23.1	.28
6	.17	− 1.2	− 68.8	.31
2	.27	+ 54.6	+ 67.1	.07
5	.32	+229.3	− 65.1	+0.19
Parallax star	0.17	− 6.00	− 22.20	.....

The mean magnitude of the comparison stars is approximately 10.2.

TABLE 2

REDUCTIONS FOR *17 Lyrae C*

Plate	Solution (m)	Weight (p)	Parallax Factor (P)	Time in Days (t)	Residual (v)	$\sqrt{p}\cdot v$ in Arc
10A.....	+0.604	0.7	+0.418	−844	+ .007	+ ″02
77.....	1.232	1.0	−0.608	−308	− .008	− .02
90.....	1.462	.7	+0.959	−196	+ .005	+ .01
105.....	1.584	.5	−0.899	− 21	− .012	− .02
245.....	1.954	.7	+0.992	+189	− .001	.00
252.....	1.978	.7	+0.781	+224	− .010	− .02
280.....	2.048	.7	−0.490	+308	+ .009	+ .02
282.....	2.050	1.0	−0.679	+322	+ .002	+ .01
288.....	+2.061	0.7	−0.749	+328	+ .008	+0.02

The normal equations are:

$$6.7c + 0.0980\mu - 0.3988\pi = +11.1489$$
$$+92.6136\mu - 4.2438\pi = +11.9000$$
$$+3.6775\pi = -1.0397$$

from which

$$c = +1.6649$$
$$\mu = +0.1289 = +0''.343 \qquad = +0^{s}.099 \text{ per year}$$
$$\pi = +0.0465 = +\mathbf{0''.124 \pm 0''.008}$$

Probable error corresponding to unit weight, $\pm 0.0052 = \pm 0''.014$.

P Cygni ($20^{h}14^{m}$, $+37°43'$)

"This previously unrecorded star was discovered by Janson on August 18, 1600. It was observed by Kepler two years later when it was of the third magnitude, and it remained visible to the eye until 1621. Cassini found it to be of the third magnitude for a short time in 1655, and, after another less-marked rise in 1665, it slowly declined in brightness. Since 1677, the light of the star has been very nearly constant, at magnitude 5.0" (Frost, *Astrophysical Journal*, **35**, 286, 1912). The spectrum, examined by Frost (*loc. cit.*), is of the B type, but peculiar in showing intense, broad, bright lines of hydrogen and helium.

Eleven plates of this star were obtained for parallax determinations as given in Table 1. They were measured by Mr. Mitchell.

TABLE 1

PLATES OF *P Cygni*

No.	Date	Hour Angle	Observers	Quality of Images
281	1910 Aug. 13	-1^h5	Su, Sl	One fair
283	Aug. 20	−0.6	Su, Sl	Good
289	Aug. 26	−0.4	Su, Su	Good
297	Sept. 9	−1.0	Su, Su	One fair
304	Oct. 1	−0.5	Sl, Sl	Good
317	Oct. 15	−0.2	Sl, Sl	Poor
428	1911 June 17	−1.6	Su, Sl	Good
513	Oct. 7	−0.6	Sl, Sl	Fair–Good
760	1912 May 16	−1.6	Su, Sl	Good
761	May 16	−1.4	Su, Sl	Good
792	June 22	+0.2	Su, Sl	Good

Plates 297 and 428 were each measured twice.

COMPARISON STARS

No.	Diameter	*X* (right ascension)	*Y* (declination)	Dependence
	mm			
1	0.18	−293	− 65	+0.168
2	.16	−136	+126	.205
4	.18	+ 65	−148	.187
5	.19	+114	− 7	.208
6	.15	+250	+ 94	+0.232
Parallax star	0.17	+ 16.8	+ 7.7	.....

The average magnitude of the comparison stars is 9.5. The rotating disk was used to cut down the light of the parallax star.

TABLE 2

REDUCTIONS FOR *P Cygni*

Plate	Solution (m)	Weight (p)	Parallax Factor (P)	Time in Days (t)	Residual (v)	$\sqrt{p}\cdot v$ in Arc
281	+.042	0.5	−0.324	−339	− .019	−0".03
283	.072	1.0	− .430	−332	+ .010	+ .03
289	.066	1.0	− .516	−326	+ .004	+ .01
297	.055	0.7	− .696	−312	− .009	− .02
304	.052	1.0	− .896	−290	− .013	− .03
317	.074	0.4	− .958	−276	+ .008	+ .01
428	.060	1.2	+ .575	− 31	+ .008	+ .02
513	.080	0.8	− .928	+ 81	+ .016	+ .04
760	.043	1.0	+ .896	+303	− .005	− .01
761	.060	1.0	+ .896	+303	+ .012	+ .03
792	+0.063	1.0	+0.495	+340	−0.018	−0.05

The normal equations are:

$$9.6c - 4.727\,\mu - 0.6398\pi = +0.5511$$
$$76.2292\mu + 15.1326\pi = -0.4242$$
$$+\ 4.9490\pi = -0.0829$$

These yield

$$c = +0.0566$$
$$\mu = -0.0004 = -0''.001 \qquad = -0^{s}.000 \text{ per year}$$
$$\pi = -0.0080 = \mathbf{-0''.021 \pm 0''.016}$$

Probable error corresponding to unit weight $= \pm 0.0087 = \pm 0''.023$.

τ Cygni ($21^{h}10^{m}$, $+37°37'$)

This is a 3.8-magnitude star discovered in 1874 to be double by Alvan G. Clark with the 26-inch of the Leander McCormick Observatory. The components *A* and *B* of 4.9 and 7.4 magnitudes are separated approximately 1″. Since the discovery, the companion has passed over 270° of position angle. The period is 55± years. The system has a proper motion, according to Boss, of 0″.455 in position angle 20°.3. 15″ distant is a fainter component.

τ Cygni is of spectral type F, and was found by Barrett to be a spectroscopic binary (*Astronomische Nachrichten*, **177**, 174, 1908). According to the above conditions, it is reasonable to expect *τ Cygni* to possess an appreciable parallax. G. Abetti has

TABLE 1

PLATES OF *τ Cygni*

No.	Date	Hour Angle	Observers	Quality of Images
112	1909 Sept. 18	$0^{h}0$	Su, Sl, Su	Fair
259	1910 June 18	−1.0	Su, Sl	Fair
268	June 25	−1.4	Su, Sl	Good
299	Sept. 9	−0.4	Su, Su	Good
328	Oct. 22	0.0	Sl, Sl	Fair
430	1911 June 17	−1.7	Su, Sl	Good
444	July 8	−0.7	Su, Sl, Sl	Good
482	Sept. 2	−0.4	Su, Sl	Good
495	Sept. 16	−0.1	Su, Sl	Good
779	1912 June 8	−0.5	Su, Sl	Good
793	June 22	−0.4	Su, Sl	Fair
794	June 22	−0.1	Su, Sl	Good
797	June 23	0.0	M, Su	One good

Plate 797 was measured twice.

COMPARISON STARS

No.	Diameter	X (right ascension)	Y (declination)	Dependence
	mm			
1	0.15	−285	−149	+0.253
2	.23	− 29	+214	.269
4	.16	+130	−243	.223
5	.20	+184	+178	+0.255
Parallax star	0.26	− 3.9	+ 11.0	.....

published (*Mem. Soc. Spettros. Italiani*, November, 1912) an interesting discussion of τ *Cygni*, in which he assumes a parallax of 0".06.

Thirteen plates of the star were obtained. These were measured by Mr. Mitchell.

The average magnitude of the comparison stars is 9.5. The rotating disk which was used to reduce the brightness of the parallax star cut out both companion stars.

TABLE 2

REDUCTIONS FOR τ *Cygni*

Plate	Solution (m)	Weight (p)	Parallax Factor (P)	Time in Days (t)	Residual (v)	$\sqrt{p} \cdot v$ in Arc
112	+0.077	0.9	−0.619	−504	+ .005	+0".01
259	.102	0.7	+ .724	−201	− .014	− .03
268	.094	1.0	+ .645	−224	− .022	− .06
299	.155	1.0	− .498	−148	+ .030	+ .08
328	.104	0.7	− .911	−105	− .026	− .06
430	.192	1.0	+ .738	+133	+ .023	+ .06
444	.183	1.2	+ .477	+154	+ .011	+ .03
482	.176	1.0	− .394	+210	− .002	− .01
495	.170	1.0	− .588	+224	− .010	− .03
779	.219	1.0	+ .816	+490	− .003	− .01
793	.234	0.7	+ .674	+504	+ .010	+ .02
794	.214	1.0	+ .674	+504	− .010	− .03
797	+0.219	0.8	+0.662	+505	−0.005	−0.01

The normal equations are:

$$12.0c + 14.418\,\mu + 2.2788\pi = +1.9921$$
$$+136.2136\mu + 13.7651\pi = +4.1753$$
$$+ 5.0641\pi = +0.5524$$

These yield

$$c = +0.1478$$
$$\mu = +0.0148 = +0''.039 \qquad = +0^{s}.012 \text{ per year}$$
$$\pi = +0.0024 = \mathbf{+0''.0065 \pm 0''.016}$$

Probable error corresponding to unit weight $= \pm 0.0117 = \pm 0''.031$.

The determinations of the parallax of τ *Cygni* have been made by meridian circle.

Belopolsky obtains $\pi = +0''.06 \pm 0''.06$
Jost obtains $\pi = +0''.125 \pm 0''.063$
Abetti obtains $\pi = +0''.029 \pm 0''.039$

Nova Lacertae ($22^{h}32^{m}$, $+52^{\circ}12'$)

Nova Lacertae was discovered by Espin, December 30, 1910. A preliminary value of its parallax was published in *Astrophysical*

TABLE 1

PLATES OF *Nova Lacertae*

No.	Date	Hour Angle	Observers	Quality of Images	Remarks
359	1910 Dec. 31	$+2^{h}8$	Sl, Sl	Fair	Telescope east
360	1911 Jan. 7	+2.8	Sl, Sl, Sl	Good	Telescope east
432	June 17	−2.0	Su, Sl	Good	
438	July 1	− .6	Su	Fair	
446	July 8	− .5	Su, Sl	Good	
498	Sept. 16	.0	Sl, Su	Good	
531	Oct. 14	− .2	Sl, Su	Good	
536	Oct. 28	− .5	Su, Sl	Good	
548	Nov. 21	− .1	Sl, Sl	Good	
563	Dec. 1	− .2	Sl, Sl	Good	
802	June 29	− .9	Su, Sl	Fair	
810	July 7	− .4	Su, M	Fair	
816	July 13	0.0	Su	Poor	

COMPARISON STARS

No.	Diameter	*X* (longitude)	*Y* (latitude)	Dependence
	mm			
1	0.32	−261.6	− 77.2	+0.13
2	.22	− 76.5	+179.8	.21
3	.21	− 65.8	− 28.2	.15
4	.30	− 20.8	+ 42.8	.18
5	.24	+ 55.3	−247.5	.11
6	.27	+369.4	+130.3	+0.22
Nova	0.37 to 0.10	+ 23.46	+ 32.17	.....

Journal, **35**, 134. Since that determination was made, the observations have been extended over one more parallactic epoch, and, in all, thirteen plates obtained, as shown in Table 1. They were measured by Mr. Slocum.

The mean magnitude of the comparison stars is approximately 9.3.

TABLE 2

REDUCTIONS FOR *Nova Lacertae*

Plate	Solution (m)	Weight (p)	Parallax Factor (P)	Time in Days (t)	Residual (v)	$\sqrt{p}\cdot v$ in Arc
359.....	0.000	0.7	−0.983	−189	−0.009	−0".02
360.....	+ .002	1.2	−0.978	−182	− .007	− .02
432.....	.024	1.0	+0.984	− 21	+ .004	+ .01
438.....	.047	.5	+1.016	− 7	+ .026	+ .05
446.....	.000	1.0	+1.012	0	− .022	− .06
498.....	.036	1.0	+0.299	+ 70	+ .013	+ .04
531.....	.046	1.0	−0.176	+ 98	+ .023	+ .06
536.....	.036	1.0	−0.405	+112	+ .013	+ .04
548.....	.001	1.0	−0.736	+136	− .022	− .06
563.....	.029	1.0	−0.838	+146	+ .006	+ .02
802.....	.012	.7	+1.015	+357	− .024	− .05
810.....	.047	.7	+1.012	+365	+ .010	+ .02
816.....	+0.023	0.3	+0.998	+371	−0.014	−0.03

The normal equations are:

$$11.1c + 8.0350\mu + 0.5053\pi = +0.2461$$
$$+35.5843\mu + 6.7885\pi = +0.3189$$
$$+7.5993\pi = +0.0569$$

from which

$$c = +0.0190$$
$$\mu = +0.0042 = +0''.011$$
$$\pi = +0.0025 = +0''.007 \pm 0''.012$$

Probable error corresponding to unit weight $\pm 0.0116 = \pm 0''.031$.

Belanowsky[1] finds, by photography, $\pi = -0''.005 \pm 0''.020$.

The following table (p. 25) gives a summary of the results. The *absolute* values of the parallaxes can be found only when we know the parallaxes of the comparison stars. At present only approximate average values are available for comparison stars of different

[1] *Mitteilungen der Nikolai-Hauptsternwarte zu Pulkowo*, **5**, pt. 4, 47.

SUMMARY OF RESULTS

Star	R.A. 1900	Decl. 1900	*B.D.* Number	Magnitude and Spectrum	Proper Motion	Relative Parallax	Probable Error	Co-ord.	No. of Plates	Probable Error of One Plate
φ Andromedae	1h 4m	+46° 43′	+46.275	4.4 B 8	0″.03	**+0″.004**	±0″.008	Long	8	±0″.019
48 Cassiopeiae	1 54	+70 25	+70.153	4.6 A2	0.06	**−0.002**	0.016	Long	10	0.036
20 Persei	2 47	+37 56	+37.655	5.7 F	0.10	**−0.012**	0.007	R.A.	13	0.018
9 Camelopardalis	4 44	+66 10	+66.358	4.4 B	0.01	**−0.032**	0.011	R.A.	10	0.018
μ Orionis	5 57	+ 9 39	+9.1064	4.2 A2	0.03	**+0.036**	0.016	R.A.	8	0.031
Groningen VII, No. 20	16 21	+48 35	.....	10.7	1.22	**+0.125**	0.012	R.A.	10	0.022
Anonymous	17 33	+18 37	.....	9.1	1.36	**+0.108**	0.011	R.A.	4	0.019
B.D. 18°3423	17 34	+18° 37	+18.3423	9.0	0.24	**+0.003**	0.004	R.A.	9	0.009
B.D. 18°3424	17 34	+18° 37	+18.3424	9.2	0.09	**+0.003**	0.008	R.A.	12	0.018
96 Herculis	17 58	+20 50	+20.3649	5.5 B	0.02	**−0.004**	0.008	R.A.	12	0.021
17 Lyrae C	19 4	+32 21	.....	11.3	1.75	**+0.124**	0.008	R.A.	9	0.014
P Cygni	20 14	+37 43	+37.3871	4.9 B4P	0.01	**−0.021**	0.016	R.A.	11	0.023
τ Cygni	21 10	+37 37	+37.4240	3.8 F	0.45	**+0.006**	0.016	R.A.	13	0.031
Nova Lacertae	22 32	+52° 12	.....	8 to 13 P	.....	**+0.007**	±0.012	Long	13	±0.031

magnitudes. From Kapteyn's table in *Groningen Publications*, No. 24, p. 15, we have

Galactic Latitude Mean Magnitude Comp. Stars	−20° to +20°	±20° to ±40°	±40° to ±90°
8.0	+0.″006	+0.″007	+0.″009
9.0	.005	.006	.007
10.0	.004	.005	.005
11.0	+0.003	+0.004	+0.004

The mean magnitude of the comparison stars for our various fields ranges from 8.9 to 10.5, so if the mean parallax of the five or six comparison stars which we have used in each case conforms to the above table of averages, the corrections necessary to reduce our *relative* parallaxes to *absolute* parallaxes will range from +0.″004 to +0.″006.

YERKES OBSERVATORY
January 1913

PRELIMINARY RESULTS OF AN ATTEMPT TO DETECT THE GENERAL MAGNETIC FIELD OF THE SUN[1]

By GEORGE E. HALE

INTRODUCTION

The investigation[2] of the magnetic phenomena of rotating bodies dates from the classic experiment of Rowland, who in 1876 succeeded in producing a magnetic field by whirling an electrically charged disk at a high velocity.[3] Maxwell had previously suggested the hypothesis that an electrified body in motion is equivalent to an electric current, and calculated the strength of the field thus obtained,[4] but definite proof of the effect was lacking prior to Rowland's demonstration.

In 1879 Perry and Ayrton showed that since points near the surface of the earth have linear velocities of rotation different from those of the interior, a magnetic field would be produced if the earth had an initial electrical charge residing on the surface.[5] The weak point in this theory of terrestrial magnetism, as Rowland pointed out in the same year, lies in the fact that the earth must be electrified to a potential of about 41×10^{15} volts to account for the observed field. As this would be sufficient to produce a spark about six million miles long in atmospheric air of ordinary density, and would enter as a serious factor in planetary perturbations, it is evident that the theory must be dropped.[6]

In 1891 and again in 1892 Schuster raised the question whether

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 71.

[2] The first results of this investigation may be found in "Preliminary Note on an Attempt to Detect the General Magnetic Field of the Sun," *Terrestrial Magnetism and Atmospheric Electricity*, **17**, 173, 1912.

[3] Rowland, "On the Magnetic Effect of Electric Convection," *American Journal of Science* (3), **15**, 30–38, 1878.

[4] Maxwell, "Treatise on Electricity," Art. 770.

[5] Perry and Ayrton, "A New Theory of Terrestrial Magnetism," *Proc. Physical Society of London*, **3**, 57, 1879.

[6] Rowland, "On Professors Ayrton and Perry's New Theory of the Earth's Magnetism," *Proc. Physical Society of London*, **3**, 97, 1879.

every large rotating mass is not a magnet.[1] Lord Kelvin also forcibly expressed his opinion that the earth's magnetism must be due to its rotation. In 1894 J. J. Thomson remarked that if atoms exert different attractions on positive and negative electricity, then a large rotating body ought to produce a magnetic field. The maximum magnetic force at the surface of a rotating sphere would be proportional to ρr^2, where ρ is the angular velocity of rotation and r the radius. Assuming the earth's magnetic force to be the maximum attainable by the rotation of a sphere the size of the earth, he calculated that the magnetic force of a sphere one foot in radius rotating one hundred times a second would be about one hundred-millionth part of that of the earth. Hence it would probably be hopeless to detect such an effect in the laboratory.[2] Sutherland applied this idea in his hypothesis of terrestrial magnetism, and pointed out that the external electric effect would be overcome by the presence within the earth of equal charges of positive and negative electricity. These he supposed to be spread over concentric spheres whose radii differ by a very small quantity.

In a paper read before the American Association on December 31, 1912, Bauer adopts a similar view. The symmetrical part of the earth's field can then be accounted for by supposing the radius of the sphere containing the positive charge to be only 0.4×10^{-8} cm smaller than that of the sphere containing the negative charge. This difference is about four-tenths of the radius of a molecule.[3] A similar result is obtained for the portion of the earth's field due to the effect of the atmosphere—the negative electrons, on the average, are somewhat farther from the earth's center than the positive electrons. If the positive and negative electrons differ in mass and possess inertia, the earth's centrifugal force may produce a spheroidal rather than a spherical distribution of the charges, thus accounting for the observed increase in the equivalent intensity of magnetization toward the equator. Assuming the sun's field to be due to the same cause as that of the earth, Bauer computes

[1] *Report, B.A.A.S.* for 1892, p. 634; *Proc. Physical Society of London*, **24**, 121, 1912.

[2] "On the Electricity of Drops," *Phil. Mag.* (5), **37**, 358, 1894.

[3] Wilson, "Structure of Atoms," *Science*, N.S., **35**, 511, 1912.

that the vertical magnetic intensity at the sun's poles is about 300 gausses.[1] This value is in close agreement with that of Schuster, who had previously shown that the magnetic intensity of the sun should be about 440 times greater than that of the earth.[2]

A very different theory, which is preferred by Schuster, rests on the probable assumption that every molecule is a magnet. If this magnetism is accounted for as the effect of the rapid revolution of electrons within the molecule, a gyrostatic action may be anticipated. That is, each molecule would tend to set itself with its axis parallel to the axis of the earth, and the earth's magnetic field would result from the combined effect of all the molecules.

This theory, like the preceding one, is not free from obvious points of weakness. Its chief advantage lies in the possibility that it may explain the secular variation of the earth's magnetism by a precessional motion of the magnetic molecules.

As a general problem of physics, the suggestion of Schuster that every rapidly rotating body may produce a magnetic field is of fundamental importance. Since it appears to be beyond the reach of experimental test, owing to the limitations of size and rotational velocity imposed by laboratory conditions, we may take advantage of astronomical phenomena, where these limitations no longer obtain. The existence of the earth's magnetism is favorable to the hypothesis, but it remains to be determined by the observation of other heavenly bodies whether such magnetic phenomena as they may exhibit are in harmony with its assumptions. The most

[1] I am indebted to Dr. Bauer for the manuscript of an abstract of this paper, which has since appeared in the *Physical Review*. See also "A Consistent Theory of the Origin of the Earth's Magnetic Field," *Journal Washington Academy of Sciences*, January 4, 1913.

[2] Schuster, "A Critical Examination of the Possible Causes of Terrestrial Magnetism," *Proc. Physical Society of London*, **24**, 127, 1912. In this paper, and in a later one by Swann ("The Earth's Magnetic Field," *Phil. Mag.*, **24**, 80, 1912) various theories of the earth's magnetism are fully discussed. Schuster rejects the hypothesis that a neutral molecule in its motion behaves as if it carried a charge, partly because of the distinction which must be made between the magnetic effects of a rotating body on a fixed magnet and on one moving with it, and also on account of the failure of this hypothesis to explain the secular variation and the lack of coincidence of the magnetic and geographical axes. I am indebted to Professor Schuster for the results of his calculations of the strength of the sun's general field, communicated to me in September 1911, before their publication.

promising opportunity is afforded by the sun, which meets many of the necessary conditions. Its great radius and angular velocity of rotation should give rise to a field more than four hundred times as intense as that of the earth. Its atmosphere contains self-luminous vapors, giving line-spectra capable of revealing the magnetic field by the Zeeman effect. Its brightness is sufficient to permit the use of the very high dispersion required to detect a field so much weaker than the fields usually employed in laboratory studies of radiation. Finally, its axial rotation and large angular diameter facilitate observation at a great number of points on its surface, while the position of its axis, allowing line displacements to be measured near both poles, enables the observer to apply the most perfect test of the Zeeman effect—the reversal of the sign of the displacement with the polarity. A very important limitation, however, must not be overlooked. We know that the sun contains free electrons, and that the positive and negative charges are definitely segregated in sun-spots and probably also in the chromosphere, where the more active negative electrons tend toward higher levels. Hence magnetic fields, local and general, may result from the motion of these charges. It is possible, nevertheless, to determine accurately the part played by free electrons in spots, and their effect on the general magnetic field may not be beyond the range of investigation.

From the standpoint of the physicist, therefore, the sun may prove of service by throwing some light on the hypothesis of the magnetism of rotating bodies. But the problem is of no less interest and importance from an astrophysical point of view. The discovery of powerful magnetic fields in sun-spots[1] indicated the possibility of a general observational attack on solar magnetic phenomena. The Zeeman effect was found to extend well beyond the limits of the penumbra, and the configuration of the hydrogen flocculi suggested that with suitable polarizing apparatus local fields, sometimes of great extent, might be detected in regions far removed from visible spots. The next logical step was the exploration of the sun's general field.

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 30; *Astrophysical Journal*, **28**, 315–343, 1908.

So far as I am aware, the only direct method of detecting the magnetic field of the sun is by the observation of the Zeeman effect. An indirect method, first applied by Bigelow to the corona in 1889, led him to infer that the sun must be a magnet because the coronal streamers, especially near the poles, agree well in form with the lines of force of a magnetized sphere.[1] Störmer has recently calculated the trajectories of electric corpuscles moving out from the sun under the influence of an assumed magnetic field, and the resulting curves closely resemble the structure of the corona.[2] Finally, Deslandres has applied the same idea in the case of the prominences, and concludes from their forms and radial velocities that the ions which compose them are moving under the influence of the sun's field.[3] The results already obtained by these methods are of extreme interest, and promise to be of even greater importance when fully developed in the future. It should be noted, however, that any conclusions thus reached as to the sun's general field must relate to the field existing at considerable elevations in the solar atmosphere, which may differ greatly in intensity, and may even be opposed in sign, to the field produced by the rotation of the body of the sun lying within the photosphere. Furthermore, since the sign of the charge of the outflowing electrons must be assumed, and since this sign may not always be the same, no certain conclusion can be reached as to the polarity of the general field or the sign of the charge that may produce it, even when the velocity of the electrons is accurately known.

These and other considerations led Schuster, when summing up the situation in April 1912, to remark: "The evidence [whether the sun is a magnet] rests entirely on the form of certain rays of the corona, which—assuming that they indicate the path of projecting particles—seem to be deflected as they would be in a magnetic field, but this evidence is not at all decisive."[4]

[1] Bigelow, "The Solar Corona," Smithsonian Institution, 1889.

[2] Störmer, "Sur la structure de la couronne du soleil," *Comptes Rendus*, February 20, 1911.

[3] See several papers by Deslandres in the *Comptes Rendus*, the chief results of which are given in "Champ magnétique général des couches supérieures de l'atmosphère solaire. Vérifications nouvelles." *Comptes Rendus*, December 30, 1912.

[4] Schuster, *op. cit.*, p. 131.

METHOD OF OBSERVATION

The observations were begun in 1908 with the 30-foot spectrograph of the 60-foot tower telescope.[1] This instrument served admirably for the strong magnetic fields (maximum strength about 4500 gausses) in sun-spots, but the polarizing apparatus was not well adapted for an investigation of extremely weak fields. Certain minute displacements of the solar lines, such as would have been caused by a general magnetic field, were found on measuring the

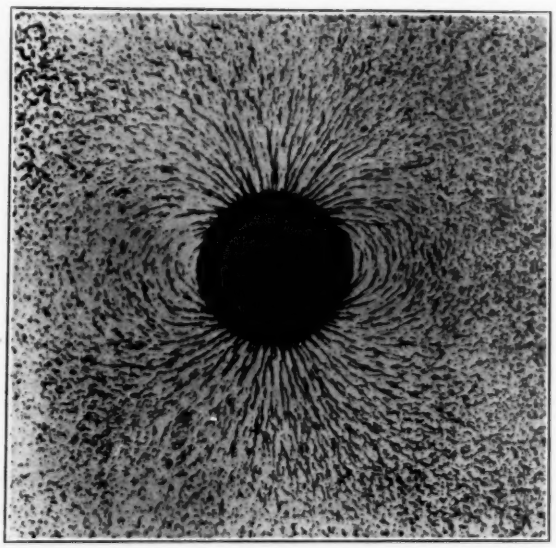


FIG. 1.—Lines of force of a magnetized sphere

photographs, but these were not such as to command confidence, and further work was deferred until better polarizing apparatus should become available. The great solar activity at that time, giving rise to strong magnetic fields in sun-spots and their neighborhood, was another adverse factor. During the present minimum of activity, sun-spots and other disturbances are rarely observed, and the unusually quiet condition of the solar atmosphere is precisely what is needed for an investigation of this nature.

[1] *Publications Astronomical Society of the Pacific*, **20**, 287, 1908.

Let us assume the sun's magnetic field to be similar to that of a magnetized sphere, with magnetic poles corresponding in position with the poles of rotation. The lines of force would then appear as in Fig. 1, the angle δ between them and the solar surface being given by the expression

$$\tan \delta = 2 \tan \phi$$

when ϕ is the heliocentric latitude. If the field were strong enough, and if the observer could look along the sun's axis and form an image of the one of the magnetic poles on the slit of a powerful spectrograph, he would find certain solar lines split into doublets, with components circularly polarized in opposite directions. If a Nicol prism were placed in front of the slit of the spectrograph with its long axis parallel to the slit, in combination with a quarter-wave plate set with its principal section at an angle of 45°, one of the components of the magnetic doublets would be extinguished while the other would be transmitted by the Nicol, as in the case of a sun-spot. Assuming the red component to be transmitted, rotation of the quarter-wave plate through an angle of 90° would cause this to be extinguished and the violet component to be transmitted. Consequently, if the quarter-wave plate were built up of mica strips 2 mm wide, mounted so that the principal sections of successive strips make an angle of 45° with the slit and 90° with each other, the Nicol would transmit (say) the red component for the odd strips and the violet component for the even strips. In a photograph of the spectrum the lines would have a dentated appearance, the magnitude of the separation of the components shown on successive strips varying directly with the strength of the field. If, from the same place of observation, the slit of the spectrograph were directed, not at the sun's pole, but at a point in 45° latitude, the effect would still be clearly observable, though the transformation of the circularly polarized light of the components into elliptically polarized light would result in less complete extinction by the Nicol.

In practice, on account of the weakness of the sun's magnetic field as compared with the fields of sun-spots, complete separation into doublets is not to be expected. Moreover, the terrestrial observer, who is close to the plane of the sun's equator, must look in

a direction nearly at right angles to the lines of force at the sun's poles. Thus he cannot take full advantage of the fact that the total intensity of the sun's magnetization is twice as great at the poles as at the equator. Fortunately, however, the angle between the lines of force and the line of sight (for an observer in the plane of the solar equator) is reduced to zero at 35° north or south latitude. But the most favorable position for observation is latitude 45°, where the effect of the ellipticity of the light of the components is overcome by the increased strength of the field.[1] It was hoped that in this latitude a spectrograph of very high dispersion might reveal slight shifts of the lines, caused by the extinction of the red and violet components by the successive strips of the quarter-wave plate. The increasing ellipticity of the *n*-components below 35°, coupled with the increasing strength of the *p*-component of triplets, and the decrease in the total intensity of the field toward the equator would be indicated by decreasing displacements of the lines. Toward the poles, for similar reasons, the displacements should also decrease.

When a source of light is between the poles of a magnet, and one of its components (observed along the lines of force) is cut off by a quarter-wave plate and Nicol, reversal of the current through the magnet extinguishes the visible component and causes the other to appear. Hence, in the case of the sun's general field, the sign of the displacements should be reversed in passing from the northern to the southern hemisphere, on account of the change of polarity.[2]

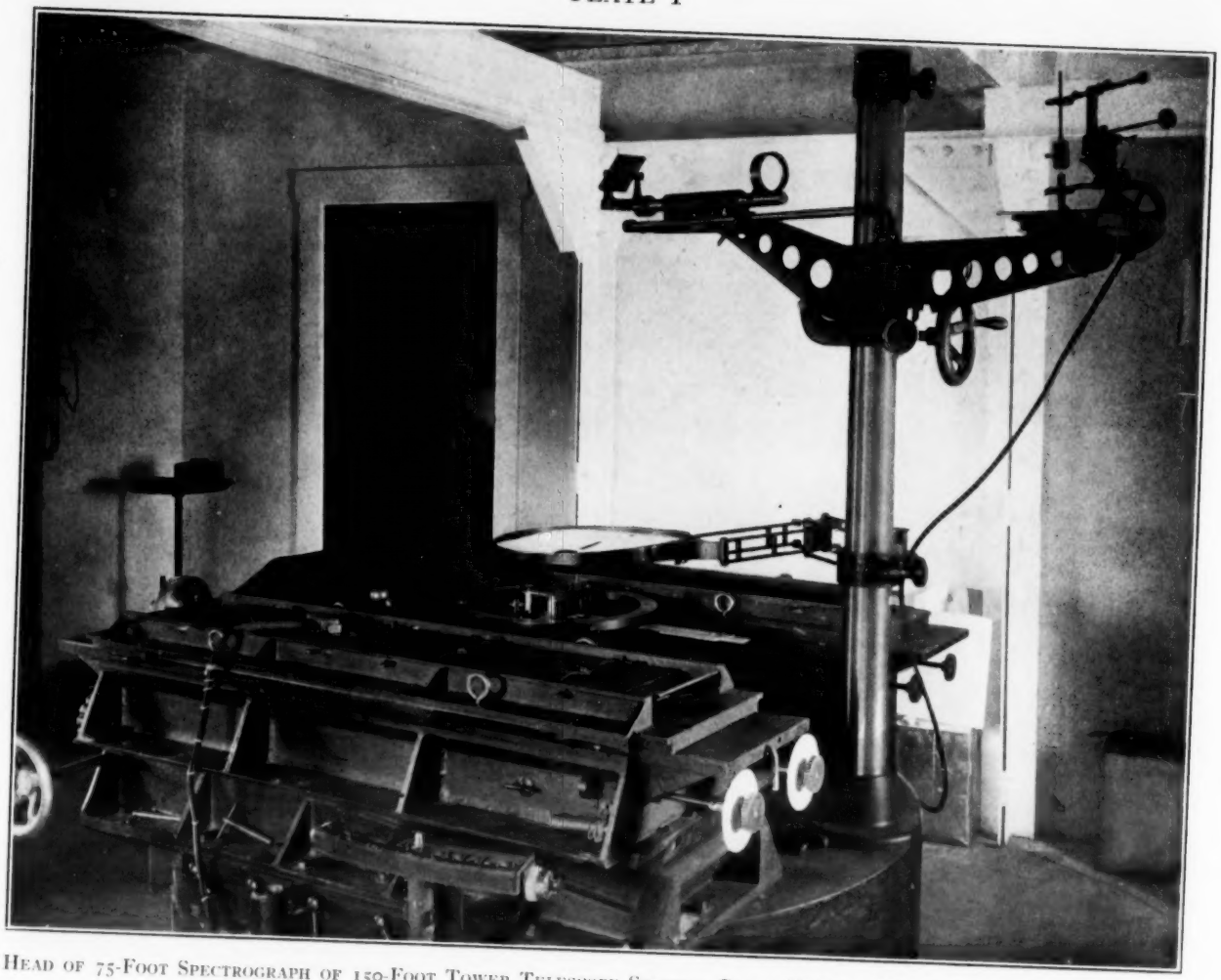
The investigation was resumed in October 1911 with the polarizing apparatus described above mounted over the slit of the 30-foot spectrograph of the 60-foot tower telescope, but no displacements which could safely be attributed to the sun's field were detected. Fortunately, the completion of the 75-foot spectrograph of the 150-foot tower telescope soon provided a much more powerful instrument, especially adapted for the purposes of this investigation.

A coelostat mounted at the summit of a tower 164 feet (50 m) in height sends a beam of sunlight to a second mirror, from which it is reflected vertically downward to an objective of 150 feet (45.7 m)

[1] See p. 83.

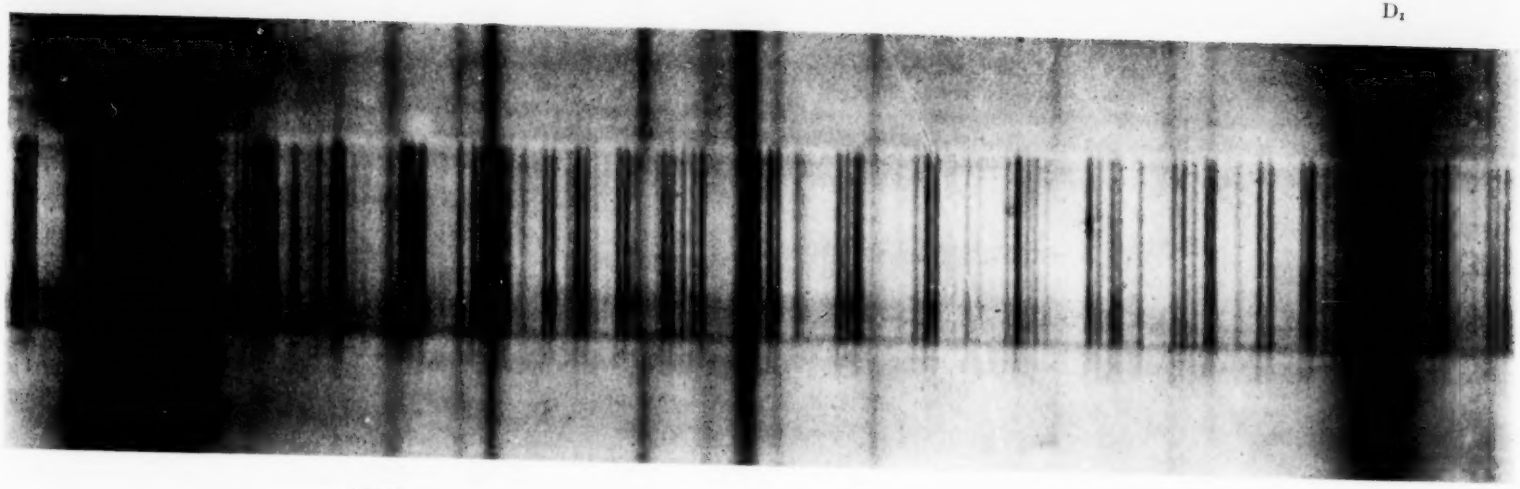
[2] Curves showing the theoretical displacements are given below (Figs. 2–7).

PLATE I

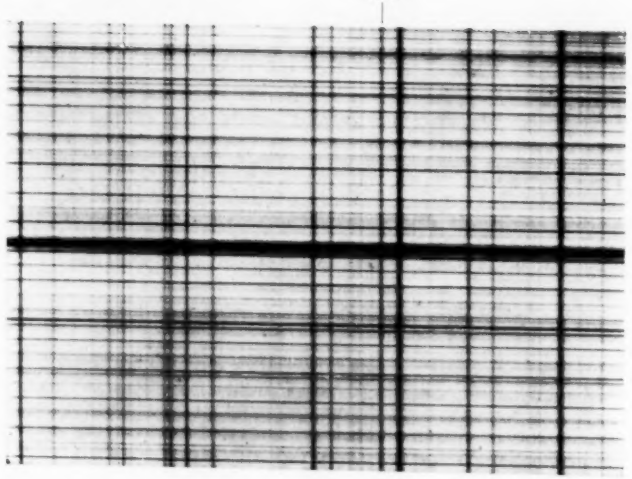


HEAD OF 75-FOOT SPECTROGRAPH OF 150-FOOT TOWER TELESCOPE SHOWING PLATE HOLDER, POLARIZING APPARATUS MOUNTED ABOVE SLIT, AND PARALLEL MOTION DEVICE USED FOR CENTERING SOLAR IMAGE

PLATE II



a



b

a. Solar spectrum and absorption spectrum of iodine vapor, region of the D lines of sodium, photographed in the third order with the 75-foot spectrograph. Slit-width, 0.127 mm. Central section, solar and iodine spectrum, exposure 35 min. Adjacent sections, solar spectrum, exposure 24 min. Scale: 1 Å = 27.8 mm. Enlargement 5.7 times.

b. Region of λ 5930 photographed in the third order with the 75-foot spectrograph showing the division of the spectrum into 2-mm strips by the compound quarter-wave plate. The heavy horizontal line marks the junction of two sections of the Nicol. The fifth strip below is the "marked strip" used for reference purposes. Scale: 1 Å = 4.90 mm.

focal length, which forms an image of the sun about 16½ inches (43 cm) in diameter in the laboratory at the base of the tower. The slit of the spectrograph, on which the image falls, is about 3 feet (0.9 m) above the floor (Plate I). After passing through the slit, the light descends to the collimating lens of 75 feet (22.9 m) focal length, mounted near the bottom of a well about 80 feet (24.4 m) deep, excavated in the earth beneath the tower. Below this lens, in a suitable mounting, is a large Michelson grating of very high resolving power (about 622 lines to the millimeter, available ruled surface 67×126 mm). After falling on the grating the light is returned through the collimating lens, which thus serves also as a camera-objective, and forms an image of the spectrum on a plate mounted close beside the slit of the spectrograph. In a single exposure a portion of the spectrum 40 inches (1 m) long can be photographed. A complete description of this instrument, which is also designed for use as a spectroheliograph, will soon be published.

The first photograph of the solar spectrum made with this spectrograph in January 1912 showed that a very high degree of precision might be expected in measurements of the positions of the solar lines. In the third-order spectrum, where much of the work described in this paper has been done, the scale at λ 5900 is 1 Ångström = 4.9 mm (Plate II, *b*). On this scale the distance between the D_1 and D_2 lines of the solar spectrum is 29 mm. An excellent test of the resolving power of the instrument is afforded by the exceedingly fine lines in the absorption spectrum of iodine, obtained by inserting a glass globe containing iodine vapor in the path of the solar beam[1] (Plate II, *a*). Near λ 5458 is a pair barely resolved photographically, having a measured separation of 0.025 Ångström. The theoretical resolution for this region, calculated from the formula

$$\frac{\delta\lambda}{\lambda}=\frac{1}{nm},$$

[1] The iodine absorption globe, for the use of which we are indebted to the suggestion of Wood, also provides a most convenient and accurate method of focusing the spectrograph. Settings made visually or photographically on the sharp iodine lines give the focus with much higher precision than would be attainable if the more diffuse lines of the solar spectrum were used.

is 0.023 Ångström. The width of the "normal slit," given by $\frac{f\lambda}{4D}$ where f is the focal length and D the aperture of the spectrograph, is 0.0288 mm. For the slit-width used in making the photograph (0.076 mm) Schuster gives a purity factor of 90 per cent.[1] Hence, on the above theory, the spectrograph should not separate lines in this region closer than 0.026 Ångström. In the fourth order fine iodine pairs near λ 5508 have separations of about 0.020 Ångström, but the solar lines are too diffuse for satisfactory measurement.

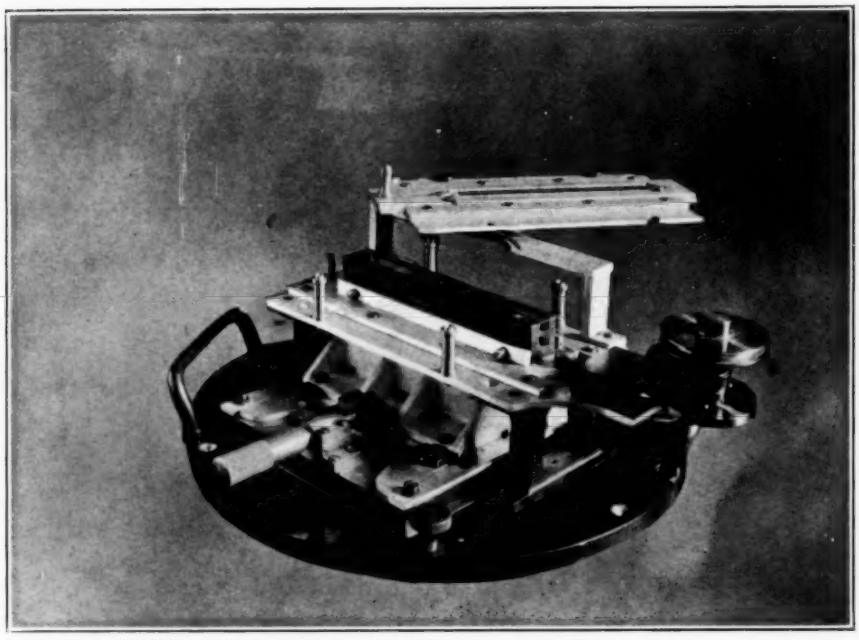
The polarizing apparatus, which forms so vital a part of the equipment required for the study of the sun's magnetic field, is illustrated in Plates III and IV. The general features of its design have been described above. The Nicol prism, 18 mm wide, built up by Werlein of four sections each 32.5 mm long so as to give a total length of 130 mm, is supported just above the slit of the spectrograph (Plate III, *a*). The junctions of the sections are necessarily marked in the spectrum by longitudinal bands (one of these appears in Plate II, *b*), but as a spectrum 90 mm wide can be photographed in a single exposure, the loss of measurable area caused by the composite structure of the Nicol is very small. If for any reason it becomes important to eliminate these longitudinal bands, this can be done by moving the Nicol prism back and forth, along the slit, during the exposure. Mechanism is provided to accomplish this, but for the purposes of the present investigation the Nicol is kept at rest, as it is important that the illumination of the grating should remain absolutely unchanged while each exposure is in progress.

For spectroscopic purposes, this long Nicol is even superior to a Nicol of the usual form, and of equal aperture (if such could be obtained). Its small thickness (8 mm) is a very decided advantage, and the impossibility of rotating it is easily overcome by the use of a half-wave plate, which can be mounted immediately above the Nicol (Plate III, *b*). Rotation of the half-wave plate through a given angle is equivalent to rotation of the Nicol through twice the angle, and the illumination of the grating is much less likely to be

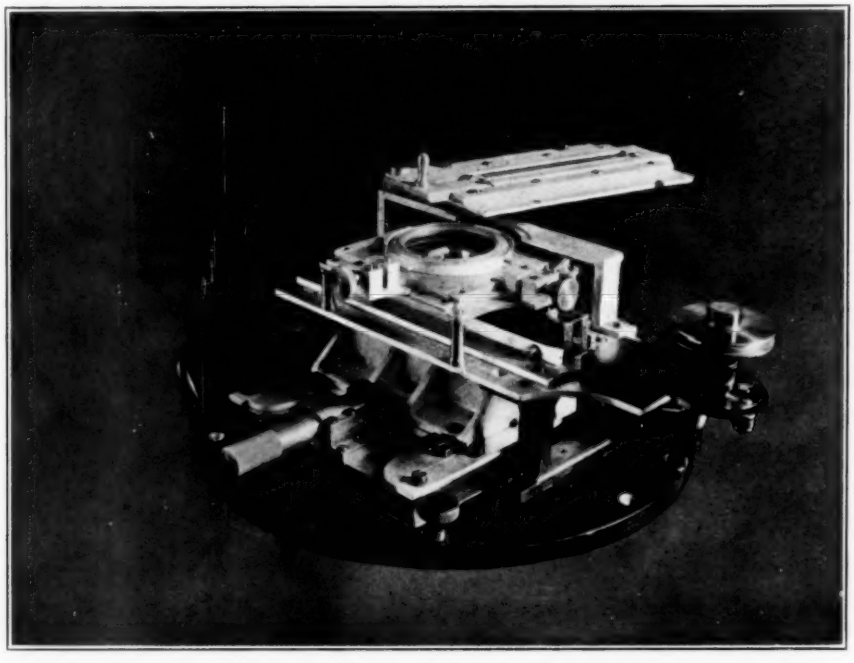
[1] *Astrophysical Journal*, **21**, 207, 1905.

PLATE III

a



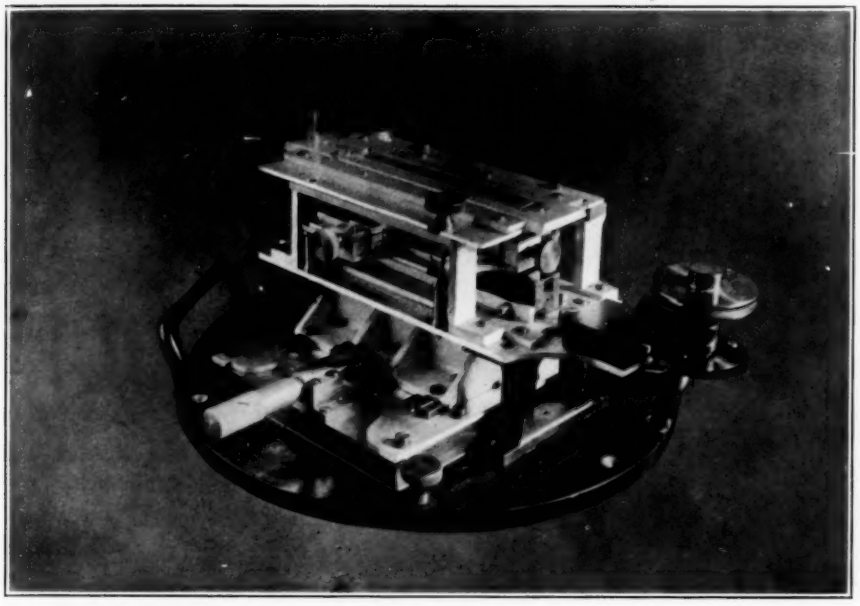
b



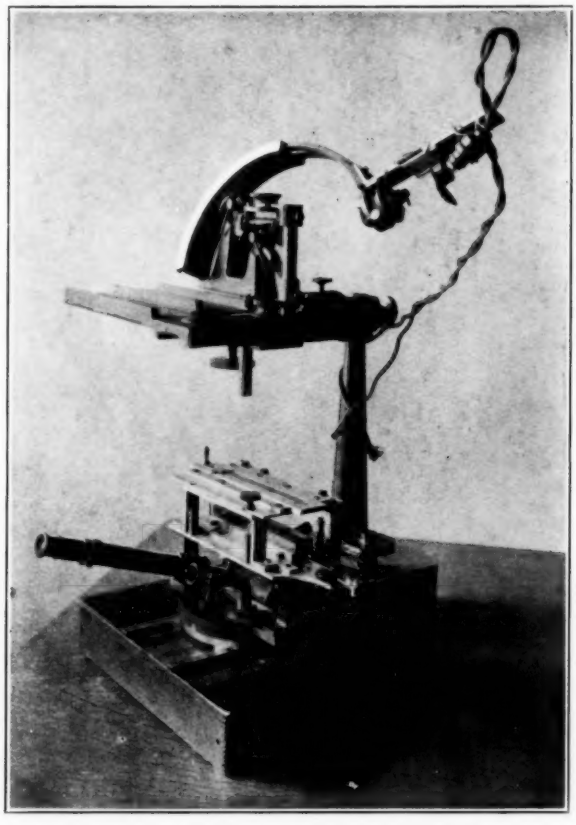
SLIT AND POLARIZING APPARATUS USED WITH THE 75-FOOT SPECTROGRAPH, COMPOUND QUARTER-WAVE PLATE SWUNG TO ONE SIDE

a. Showing the long Nicol prism mounted immediately above slit.
b. Showing the circular half-wave plate in position above the Nicol.

a



b



a. Slit and polarizing apparatus of 75-foot spectrograph with compound quarter-wave plate in position for observation.

b. Polarimeter used for measuring the elliptical polarization produced by the coelostat mirror and second flat. The polarizing apparatus is mounted on the base of the instrument in position for tests.

affected when only the half-wave plate is turned. As this arrangement is designed mainly for use in the study of the magnetic fields of sun-spots, the aperture of the (circular) half-wave plate is only 50 mm, but a half-wave plate covering the whole length of the Nicol might easily be substituted for the smaller one. In place of the half-wave plate, or in conjunction with it, circular quarter-wave plates, or half-wave and quarter-wave plates divided along a diameter into halves with their principal sections intersecting at an angle of 90°, may be employed. Such combinations are required for certain studies of sun-spot phenomena, but are unnecessary in the present work. An important experiment with the aid of the single half-wave plate, however, is described on pp. 75–76.

The compound quarter-wave plate, used for the investigation of the sun's magnetic field in conjunction with the Nicol, usually without the interposition of the half-wave plate, is shown in position for use (Plate IV, *a*). The support in which it is mounted is so pivoted that it can be swung to one side (as in Plate III) when adjustments of the Nicol or half-wave plate are to be made. On account of the great focal length of the spectrograph, the distance of the compound quarter-wave plate from the Nicol (33 mm) is not sufficient to reduce materially the sharpness of the dividing lines between adjoining strips of spectra, as the photograph reproduced in Plate II, *b* indicates.[1]

The procedure followed in making photographs of spectra for the study of the sun's magnetic field is as follows: The Nicol prism and compound quarter-wave plate are mounted above the slit, and the grating is observed to see whether the illumination from the solar image is central and complete. A parallel-motion device for orientation and guiding, supported by a massive iron column rising from the spectrograph head, is then swung into position over the slit. This carries a glass disk, on which is engraved a circle, the axis of which is to coincide with the solar axis, while the equator

[1] Dust on the slit, or small defects in the quarter-wave plate or Nicol, produce horizontal lines in the spectrum, which may be confused with the lines corresponding to the edges of the quarter-wave strips. The latter are 2 mm apart in the reproduction, which is of the same size as the original negative.

and parallels provide a scale for the purpose of setting a certain "marked strip," near the center of the compound quarter-wave plate, at any desired latitude. The disk is first rotated in its support until the axis of the circle coincides accurately with the slit. The solar image is then moved in the direction of the diurnal motion by means of the electric quick motion of the coelostat,[1] and the spectrograph head is rotated until the sun's limb runs along the equator (drawn as a straight line at right angles to the axis) of the circle. The azimuth of the slit is given by a large position circle, reading by vernier to 1′, engraved on the circumference of the spectrograph head. The tabulated position angle of the sun's axis for the date of observation is then the angle through which the spectrograph head must be turned in order to make the slit parallel to the sun's axis. The average error of orientation by this method does not exceed 15′, and could easily be made less, if this were necessary.

The grating support at the bottom of the well is not connected with the spectrograph head, but mounted on an independent support, which is rotated in azimuth by means of an electric motor until a telescope on the spectrograph head indicates the correct circle reading. The lines of the grating are then parallel to the slit. The grating itself is next rotated by another electric motor, about an axis parallel to the rulings, until the desired part of the spectrum comes into view. For most of the work described in this paper the region of the second or third order including the lines λ 5812, λ 5828, and λ 5930 was the one photographed. The overlapping spectra are cut out by a screen which transmits the region less refrangible than λ 5800. If the illumination of the grating is still perfect, and the focal setting of the collimator-camera objective (obtained with the greatest precision by the aid of the iodine absorption spectrum) is adjusted (by electric motor), the spectrograph is ready for the exposure as soon as the plates are inserted. Seed "23" or Seed "Process" plates, of very fine grain, sensitized for the red by Wallace's method, are used for the region below λ 5800. It now remains to set the solar image in the proper position on the slit.

[1] Careful tests, made from time to time, show that this direction corresponds closely with that obtained by allowing the image to drift.

The orientation device is moved until the shadow of a wire, crossing the axis of the circle at the desired latitude, falls on the "marked strip" of the quarter-wave plate. This is easily accomplished without destroying the coincidence of the axis with the slit, by the aid of the parallel-motion support. It is then only necessary to move the solar image, with the electric motions of the coelostat and second mirror, until it lies centrally within the circle, and to keep it there throughout the exposure. This amounts to about 20 minutes at λ 5900 in the third order, with a Seed "23" plate. The stability of the spectrograph, the constancy of temperature at the grating level, and the optical perfection of the air within the well are sufficiently indicated by the high resolution of the iodine absorption spectrum (Plate II, *a*) photographed in the third order on a "Process" plate with an exposure of 35 minutes.

At λ 5900 in the third order the effective aperture of the spectrograph is 126 mm, while its focal length is 22.9 m. Hence the ratio is 1/181. In the tower telescope two visual objectives, each of 12 inches (30.5 cm) aperture, but of different focal lengths, have been used to form the solar image on the slit. For the first of these the focal length is 60 feet (18.3 m), and the ratio 1/60. The corresponding diameter of the solar image is about 17 cm. The second objective has a focal length of nearly 150 feet (45.7 m), and gives a solar image about 43 cm in diameter. The ratio of 1/150 indicates that the circle of light from a short slit is 152 mm in diameter on the spectrograph objective (taking no account of diffraction), and hence more than sufficient to fill the grating. An objective of 30 feet (9.1 m) focal length was also used for a time, but the solar image was too small for satisfactory measures in the higher latitudes.

It will be observed that the greatest source of error in the comparison of the spectra of different light-sources, namely the displacement of lines caused by differences in the illumination of the grating, is not to be feared in the present investigation. Nevertheless, although the method is purely differential, the adjustments were made with the same care needed in other classes of work. Various means of checking the results are described in the discussion of possible sources of error (p. 63). The most valuable of these

is the practice of making duplicate exposures with the compound quarter-wave plate in the normal (+) and inverted (−) positions which should give displacements of opposite sign if they are caused by a magnetic field (p. 77).

The preliminary selection of solar lines for measurement was determined by three principal considerations:

1. The less refrangible region of the spectrum is advantageous because, on the average, the separation of the components of lines by a magnetic field varies directly as the square of the wave-length.

2. However, too great a wave-length is undesirable, since the average sharpness of the solar lines decreases as the wave-length increases.

TABLE I

MEASURES BY MISS LASBY OF PLATE T′3, LAT. S. 50°

(January 5, 1912, 3^h48^m–4^h25^m, P.S.T.)

λ	Origin	Intensity	$\frac{\Delta\lambda}{\lambda^2}$	Displacement
				mm
5884.028.....	*Fe*	4	1.259	+0.002
5884.120.....	*A*	5	.....	+ .001
5892.608.....	*A*	3	.....	.000
5892.920.....	*Fe*	∞	.....	− .008
5893.097.....	*Ni*	4	.....	− .002
5893.268.....	*A*	0	.....	.000
5899.215.....	*A*	2	.....	− .001
5899.518.....	*Ti*	1	1.885	− .002
5900.135.....	*A*	2	.....	− .001
5915.840.....	*A*	1	.....	+ .001
5916.475.....	*Fe*	3	.....	− .005
5916.800.....	*A*	0	.....	− .001
5918.773.....	*Ti*	0	2.518	+ .001
5928.013.....	*Fe*	2	.....	− .002
5928.510.....	*A*	2	.....	+ .001
5929.898.....	*Fe*	2	.....	− .013
5930.406.....	*Fe*	6	1.669	+ .001
5941.845.....	*A*	2	.....	.000
5941.985.....	*Ti*	∞	2.563	+ .001
5942.789.....	*A*	3	.....	.000
5952.943.....	*Fe*	4	.....	+ .002
5953.386.....	*Ti*	1	1.798	.000
5975.330.....	*A*	1	.....	+ .001
5977.007.....	*Fe*	4	1.777	−0.001

3. The large separations $\left(\frac{\Delta\lambda}{\lambda^2}\right)$ observed in the laboratory for certain spark lines indicated that these should be tried if sharp enough in the sun. However, it might easily happen that other

solar lines, too weak in laboratory sources to give the Zeeman effect, would prove to have larger displacements.

These considerations led to the selection of the region λ 5800–λ 6000, in which the lines shown in Table I are found.

The tabulated values give the double displacement, corresponding to the separation of the *n*-components of a Zeeman triplet.[1]

The measured displacements of most of the solar lines in Table I are too small to be regarded as genuine. But the iron[2] line λ 5929.898, giving a displacement of −0.013 mm (corresponding to nearly 0.003 Ångström), is apparently in a different class. An examination with a hand magnifier of the less refrangible region of Plate T′4 (Lat. S. 50°, January 6, 1912, 9^h0^m–9^h27^m P.S.T.) showed three other lines having clearly visible displacements. As measured by Miss Lasby, the displacements of these lines on two adjoining sets of strips (four strips in each set) were as follows:

TABLE II

λ	Origin	Intensity	First Set	Second Set
			mm	mm
5812.139.....	*Fe*	0	−0.012	−0.013
5828.097.....	..	0	−0.011	−0.014
5831.821.....	*Ni*	1	−0.007	−0.006

For the preliminary investigation it was thought best to obtain a large number of measures of a few lines, rather than a smaller number of measures of many lines. If the results were found to indicate beyond reasonable doubt that the displacements should be attributed to the Zeeman effect, the inclusion of other lines would naturally follow. The three lines showing the largest displacements, λ 5812.139 (*Fe*, 0), λ 5828.097 (—, 0), and λ 5929.898 (*Fe*, 2) were accordingly selected for systematic investigation at various latitudes in both hemispheres of the sun.

[1] This double displacement, however, does not give a direct measure of the strength of the solar field, owing to the increasing ellipticity toward the poles and equator of the light of the components, which involves less and less complete extinction by the quarter-wave plate and Nicol. See p. 91.

[2] Recent results indicate that this line may not be due to iron, though it is so identified by Rowland.

RECORD OF OBSERVATIONS

Two hundred and eighty-eight photographs of spectra have been made for the purposes of this investigation, but only those which have been measured and discussed are included in the present record. A considerable number of plates have been rejected on account of photographic defects. From the earlier measures it was found that there is a close connection between discordances in the measured displacements and what ordinarily would be regarded as only small deviations from high technical quality in the photographs. Thereafter each plate was carefully scrutinized, and if it presented any imperfection it was rejected without measurement. This exclusion was even more rigorously exercised with the later series than with the earlier, and doubtless accounts in part for a considerable gain in precision.

The first observations were made in January 1912, immediately after the completion of the 75-foot (22.9 m) spectrograph. As already stated, negative results had been obtained in the previous autumn with the 60-foot tower telescope and 30-foot (9.1 m) spectrograph, and as the long-focus objective of the new tower had not been completed, the 12-inch (30.5 cm) objective of 60 feet (18.3 m) focal length was temporarily transferred from the old to the new tower, and used for the first series of observations. This extended from January 6 to February 29, 1912.

Many photographs were made during March and April of the λ 5930 region in the second order and in the blue of the third order, where the line λ 4446.566 had seemed to show displacements. The objective of 30 feet focal length was used to form the solar image on the slit, but the rapid change of latitude in the higher zones rendered so small an image unsuitable. As the measured displacements of λ 5930 on these plates proved to be much smaller than in the case of the third-order plates, another series of observations in the second order (called here the second series) was begun on May 30 with the objective of 150 feet focal length, which had just been received from Brashear. At this period the temperature of the coelostat and second mirrors was not controlled, and the solar image showed large changes of focus and marked astigmatism. On July 17 the lower section of the spectrograph was raised from the

bottom of the well to the 30-foot level, so as to permit a collimator-camera objective of 30 feet focal length to be used for photographing the spectrum of the chromosphere. As the difficulty with astigmatism continued, a steady flow of water from a large tank was maintained in water jackets surrounding the sides and backs of the two mirrors. This completely cured the astigmatism and greatly reduced the range of focal length. On November 21, after the lower section of the spectrograph had been replaced at the bottom of the well, observations of the sun's field were resumed with the spectrograph of 75 feet focal length (third order) and the 43-cm solar image.

From November 21 to 29 the observations, with few exceptions, were on λ 5929.898. These are designated as the third series. The fourth series includes the results for $\lambda\lambda$ 5812.139 and 5828.097 obtained between the dates January 29 and February 18, 1913. The photographs of the second, third, and fourth series were made in pairs (quarter-wave plate in the normal and inverted positions) to provide an additional test of the magnetic nature of the displacements.

The following data are recorded in the observing book: plate number, date, hour, and minute of beginning and end of exposure, hemisphere (northern or southern), slit-width, order, region and focus of spectrum, focus of solar image, reading of spectrograph position circle, position of plate (which can be moved parallel to itself, so as to allow two exposures of a spectrum 90 mm wide to be made on one plate), inclination of plate, Nicol and quarter-wave plate used, position of quarter-wave plate (normal or inverted), distance of "marked strip" from center of sun, position angle of half-wave plate (if used), ray filter and kind of plate used, quality of seeing, telescope objective used, spectrograph objective used, grating used, temperature of well, temperature of observing room, and position of coelostat. During morning observations the coelostat stands at 600 mm east, in the afternoon at 600 mm west. Its position in a north-and-south direction with reference to the (fixed) second mirror varies with the declination of the sun. Both readings should be known, as they affect the orientation of the spectrograph, and must agree with the settings of the small

polarimeter (p. 70) used to reproduce and measure the polarization phenomena of the tower telescope mirrors corresponding to a given declination and hour angle of the sun.

For the present investigation, however, only a part of these data is required. The details of importance for the four series of observations here discussed are collected in Table III, pp. 46–53. The latitudes for which the displacements have been measured were found by means of a scale so graduated as to read directly the required values. The recorded latitude of the "marked strip" afforded the necessary data for the adjustment of the zero point of the scale. The value of λ in the fifth column is the wave-length for which the quarter-wave plate used is corrected. The signs in the sixth column refer to the position of the plate, + indicating the normal and − the inverted position. The significance of the data for the half-wave plate in the last column is explained in the discussion of the observations (p. 76). For the first two series the value of the slit-width was usually 0.004 in. (0.102 mm); for the last two, 0.005 in. (0.127 mm).

MEASUREMENT AND REDUCTION OF THE PHOTOGRAPHS

FIRST SERIES

As already explained, the sections of the compound quarter-wave plate, each 2 mm wide, divide the spectrum longitudinally into strips of the same width (Plate II. *b*). If the red and violet components of a solar line are reduced in intensity or are cut off on successive strips, relative displacements of the lines must result, which are measured in the following manner. Four strips are used in each set of measures and four settings are made, in both the direct and reversed positions of the plate, on the solar line in each strip.[1] If we call the mean settings for successive strips a, b, c, d, the results are combined as follows, to eliminate any error due to lack of parallelism of the micrometer wire and the solar line:

$$\Delta = \frac{1}{2}\left\{\left(\frac{a+c}{2}-b\right)-\left(\frac{b+d}{2}-c\right)\right\}.$$

[1] In the earlier work all of the strips except the one being measured were covered, but later this was found to be unnecessary.

The position of the solar line on a certain strip (the "marked strip") of the quarter-wave plate is taken as the zero of reference. Displacements of the line on the adjoining strips are called positive when toward the red, and negative when toward the violet. When the line of sight is parallel to the lines of force, the strips adjoining the "marked strip" used in combination with the Nicol transmit light from the left-handed circularly polarized component. When inclined to the lines of force, the vibrations received from the outer components will be elliptical, and a part of the right-handed component will also be transmitted; but the amount will always be less than that from the left-handed component. A positive displacement, therefore, means that the light of the red component of the solar line is circularly or elliptically polarized in the left-handed direction.

As a check on the results, at least one atmospheric line was measured on most of the plates of the first series; such a line, being produced in the earth's atmosphere, should of course show no displacements. At present, atmospheric lines are measured only occasionally, as they are invariably found to give no shifts exceeding the errors of measurement.

Miss Lasby's measures of Plates 3 and 4 have already been given (Tables I and II). These were made by the above method on a Toepfer measuring machine having a 150-mm screw with a pitch of 0.5 mm. An investigation of this screw indicates very small periodic errors, amounting at the maximum to 0.0003 mm, and therefore much below the errors of setting. The errors of run are small, but in any case they would not affect small differential measures of this nature. A Gaertner machine, used for a few of the measures, has periodic errors as great as 0.002 mm, but as this corresponds to a half-revolution of the screw (0.250 mm), while the maximum displacements of the solar lines in the present work never exceed 0.020 mm, the resulting error of 0.00015 mm is quite inappreciable.

An examination of Tables I and II certainly suggests that the measured displacements of at least three of the solar lines are genuine, and not due to any fault of the spectrograph or the polarizing apparatus. A valuable check is afforded by the atmospheric

TABLE III

Record of Observations

FIRST SERIES—THIRD-ORDER SPECTRUM, λλ 5812, 5828, AND 5930

FOCAL LENGTH OF TELESCOPE OBJECTIVE 60 FEET

Plate No.	Date	P.S.T.	Latitudes Measured	Quarter-Wave Plate		Observer	Remarks
				λ	Pos.		
	1912						
T′ 4.....	Jan. 26	$9^h\ 0^m - 9^h\ 27^m$	S 43, 50, 60	6300	+	H and E	
10.....	30	8 50 — 9 15	N 52, 40	6300	+	H and E	
11.....	30	9 32 — 9 47	S 40, 52	6300	+	H and E	
14.....	31	9 55 —10 30	S 46, 55, 65	6300	+	H and E	
15.....	31	11 8 —11 28	0; S 6, 12	6300	+	H and E	
17.....	31	2 43 — 3 8	N 40, 34, 28	6300	+	H and E	
18.....	31	3 29 — 3 59	S 45, 53, 63	6300	+	H and E	
19.....	31	4 14 — 4 44	S 7, 12, 18	6300	+	H and E	
21.....	Feb. 1	.....	N 6; S 2, 12, 20, 35	6300	+	H and E	
23.....	8	3 59 — 4 20	N 60, 43, 30	6300	+	H and E	
24.....	9	11 31 —11 51	N 45, 38, 28	6300	+	H and E	
25.....	9	12 5 —12 26	N 60, 43, 33	6300	+	H and E	
26.....	16	3 14 — 3 49	S 47, 57, 74	6300	+	H and E	λ/2 Plate at 0°
27.....	16	4 8 — 4 43	S 46, 56	6300	+	H and E	λ/2 Plate at −45
28.....	16	4 53 — 5 23	S 46, 55	6300	+	H and E	λ/2 Plate at −22½
29.....	17	10 37 —11 4	N 44, 37, 29	6300	+	H and E	λ/2 Plate at 0
30.....	17	11 22 —11 47	N 37, 30	6300	+	H and E	λ/2 Plate at −45
31.....	17	11 59 —12 24	N 42, 32	6300	+	H and E	λ/2 Plate at −22½
32.....	19	2 5 — 2 35	N 42	6300	+	H and E	λ/2 Plate at 0
33.....	19	3 3 — 3 33	N 42	6300	+	H and E	λ/2 Plate at −22½
34.....	19	3 56 — 4 26	N 40	6300	+	H and E	λ/2 Plate at −45
35.....	20	2 39 — 3 44	N 45	6300	+	H and E	λ/2 Plate at ±180
36.....	20	4 0 — 4 45	N 50	6300	+	H and E	λ/2 Plate at −135
37.....	21	9 33 —10 8	N 50, 40	6300	+	H and E	λ/2 Plate at +135
39.....	21	1 53 — 2 43	N 50, 38	6300	+	H and E	λ/2 Plate at ±180

TABLE III—*Continued*

T′ 40.....	Feb. 21	3 14 — 3 59	N 55, 45, 39	6300	+	H and E
41.....	21	4 20 — 5 7	S 33, 43	6300	+	H and E
42.....	22	2 26 — 3 11	N 58, 45, 35, 26, 18	6300	+	H and E
43.....	22	3 52 — 4 52	S 36, 45, 60	6300	+	H and E
44.....	25	3 44 — 4 20	N 55, 45, 35, 25, 16, 8, 0	6300	+	H and E
45.....	25	4 49 — 5 56	S 2, 10, 17, 26, 35	6300	+	H and E
46.....	26	9 42 —10 17	N 54, 42, 32, 22	6300	+	H and E
48.....	27	9 27 — 9 49	S 27, 36, 47, 60	6300	+	H and E
49.....	27	10 23 —	N 45, 37, 27, 18, 10, 3; S 5	6300	+	H and E
50.....	28	9 7 — 9 32	S 10, 17, 26, 36, 45	6300	−	H and E
51.....	28	9 46 —10 11	N 28, 19, 12, 5	6300	−	H and E
55.....	29	9 56 —10 26	N 58, 45, 35, 25, 16, 8	6300	+	E

SECOND SERIES—SECOND-ORDER SPECTRUM, λλ 5812, 5828, AND 5930
FOCAL LENGTH OF TELESCOPE OBJECTIVE 150 FEET

125*a*.....	June 3	1 27 — 1 47	N 60, 37	5650	+	E
125*b*.....	3	1 49 — 2 9	N 60, 37	5650	−	E
126*a*.....	3	2 12 — 2 32	N 60, 37	5650	−	E
126*b*.....	3	2 34 — 2 55	N 60, 37	5650	+	E
127*a*.....	3	3 0 — 3 20	N 58, 38	5650	+	E
127*b*.....	3	3 21 — 3 42	N 58, 38	5650	+	E
129*a*.....	4	9 35 — 9 53	N 58, 38	5650	−	E
129*b*.....	4	9 55 —10 15	N 58, 38	5650	+	E
130*a*.....	4	10 23 —10 43	N 56, 38	5650	+	E
130*b*.....	4	10 45 —11 5	N 56, 38	5650	−	E

TABLE III—*Continued*

Plate No.	Date	P.S.T.	Latitudes Measured	Quarter-Wave Plate		Observer	Remarks
				λ	Pos.		
	1912						
T′ 131*a*	June 4	11^h11^m —11^h29^m	N 55, 34	5650	−	E	
131*b*	4	11 31 —11 49	N 55, 34	5650	+	E	
133*a*	4	2 9 — 2 29	S 34, 55	5650	−	E	
133*b*	4	2 37 — 2 57	S 34, 55	5650	+	E	
134*a*	4	3 3 — 3 25	S 34, 55	5650	+	E	
134*b*	4	3 25 — 3 48	S 34, 55	5650	+		
135*a*	11	2 59 — 3 39	S 32, 53	5650	+	E	
135*b*	11	3 52 — 4 40	S 32, 53	5650	−	E	
137*a*	15	4 31 — 4 52	N 62, 36	5650	−	K	
137*b*	15	4 55 — 5 16	N 62, 36	5650	+	K	
142*a*	17	10 33 —11 3	S 35, 62	5650	+	K	Spot at east limb
142*b*	17	11 6 —11 36	S 35, 62	5650	−	K	Small spot at S 10° W 4°
143*a*	18	8 14 — 8 39	N 63, 38	5650	−	K	Small spot has
143*b*	18	8 42 — 9 7	N 63, 38	5650	+	K	disappeared
144*a*	18	9 17 — 9 37	S 41, 60	5650	+	K	
144*b*	18	9 42 —10 2	S 41, 60	5650	−	K	
145*a*	18	10 15 —10 35	S 2	5650	−	K	
145*b*	18	10 38 —10 58	S 2	5650	+	K	

TABLE III—*Continued*

T′ 146*a*	June 19	1 52 — 2 15	N 43	5650	+	K	
146*b*	19	2 18 — 2 43	N 43	5650	−	K	
147*a*	19	2 53 — 3 11	S 31, 53	5650	−	K	
147*b*	19	3 14 — 3 32	S 31, 53	5650	+	K	
148*a*	19	3 45 — 4 7	N 10; S 5	5650	+	K	Spot at S 7° E 38°
148*b*	19	4 12 — 4 36	N 10; S 5	5650	−	K	
149*a*	20	9 21 — 9 56	N 53, 42	5650	+	K	Spot at S 7° E 31°
149*b*	20	10 5 —10 30	N 53, 43	5650	−	K	Interrupted for 10^{m} by failure of coelostat
153*a*	21	2 12 — 3 6	N 10; S 5	5650	+	K	Spot at E 14°
153*b*	21	3 8 — 4 15	N 10; S 5	5650	−	K	
154*a*	25	8 22 — 8 52	N 60, 35	5650	−	E	
154*b*	25	8 55 — 9 30	N 60, 35	5650	+	E	
155*a*	25	9 40 —10 5	N 11; S 4	5650	+	E	
155*b*	25	10 9 —10 33	N 11; S 4	5650	−	E	
156*a*	25	10 42 —11 7	S 31, 51	5650	−	E	
156*b*	25	11 9 —11 33	S 31, 51	5650	+	E	
159*a*	26	10 30 —10 50	N 7; S 4	5650	−	E	
159*b*	26	10 51 —11 11	N 7; S 4	5650	+	E	
160*a*	26	11 17 —11 38	S 41	5650	+	E	
160*b*	26	11 39 —11 60	S 41	5650	..	E	

TABLE III—*Continued*

THIRD SERIES—THIRD-ORDER SPECTRUM, λ 5930
FOCAL LENGTH OF TELESCOPE OBJECTIVE 150 FEET

Plate No.	Date	P.S.T.	Latitudes Measured	Quarter-Wave Plate		Observer	Remarks
				λ	Pos.		
	1912						
T′ 181*a*	Nov. 22	8^h47^m — 9^h33^m	N 46, 42, 37	5650	−	H and E	
181*b*	22	9 37 —10 22	N 62, 53, 46, 42	5650	+	H and E	
184*a*	23	8 5 — 8 45	N 60, 53, 45	5650	+	H and E	
184*b*	23	8 51 — 9 33	N 60, 53, 45	5650	−	H and E	
185*a*	23	10 28 —11 8	S 34, 38, 47, 52	5650	−	E	
185*b*	23	11 10 —11 50	S 34, 38, 47, 52	5650	+	E	
187*a*	24	8 8 — 9 0	S 33	5650	−	E	
192*b*	25	2 0 — 3 0	N 43, 39	5650	−	E	
193*a*	26	8 50 — 9 55	N 43, 39	5650	+	E	
193*b*	26	9 57 —10 57	N 60, 43, 39	5650	+	E	
194*a*	26	11 6 —12 3	S 36, 42	5650	+	E	
196*a*	27	11 2 —12 2	S 33, 38, 47, 52	5650	−	E	
196*b*	27	1 22 — 2 32	S 33, 47, 52	5650	−	E	
200*a*	29	10 17 —10 57	N 59, 52, 43, 41, 36, 32	5650	+	E	
200*b*	29	11 0 —11 40	N 58, 53, 48, 43, 35	6300	+	E	
203*a*	Dec. 5	1 39 — 2 27	N 11, 7, 2; S 2, 6	6300	−	E	
203*b*	5	2 28 — 3 30	N 11, 7, 2; S 2, 6	6300	−	E	
211*a*	14	10 7 —11 4	S 33, 37, 43, 48				

TABLE III—*Continued*

T′ 212*a*	Dec. 18	8 41	— 9 46	S 33, 37, 43, 47, 53	6300	+	E
212*b*	18	9 50	—10 51	N 59, 54, 46, 42, 37	6300	+	E
217*a*	20	8 25	— 9 37	N 51, 47	6300	+	E
217*b*	20	9 39	—10 43	N 51, 47	6300	−	E
220*a*	21	8 32	— 9 38	S 33, 45, 50, 53	6300	+	E
220*b*	21	9 40	—10 43	S 33, 38, 45, 50, 53	6300	−	E
223*a*	22	8 25	— 9 36	N 54, 45, 41, 36	6300	+	E
223*b*	22	9 37	—10 42	N 59, 54, 45, 41, 36	6300	+	E
226*a*	23	8 21	— 9 30	S 38, 45, 49, 55	6300	+	E
226*b*	23	9 31	—10 36	S 38, 45, 49, 55	6300	−	E
230*a*	28	9 24	—10 30	N 66, 59, 53, 49, 45, 39	6300	+	E
230*b*	28	10 32	—11 35	N 66, 59, 53, 49, 45, 41	6300	−	E
232*a*	29	8 47	— 9 49	S 37, 39, 47, 52, 57	6300	−	E
232*b*	29	9 50	—10 50	S 37, 39, 47, 52, 57	6300	+	E
233*a*	29	10 52	—11 57	S 37, 39, 47, 52, 57	6300	+	E
233*b*	29	1 5	— 2 14	N 56, 52, 43, 39, 35	6300	+	E

TABLE III—*Continued*

FOURTH SERIES—THIRD-ORDER SPECTRUM, λλ 5812 AND 5828
FOCAL LENGTH OF TELESCOPE OBJECTIVE 150 FEET

Plate No.	Date	P.S.T.	Latitudes Measured	Quarter-Wave Plate		Observer	Remarks
				λ	Pos.		
	1913						
T′ 255*a*	Jan. 29	10^h56^m —11^h24^m	S 49, 52, 60, 66, 73	5650	−	E	
255*b*	29	11 27 —11 55	S 47, 52, 60, 66, 72, 79	5650	+	E	
258*a*	30	8 34 — 8 58	S 42, 46, 52, 56, 63	5650	+	E	
258*b*	30	9 0 — 9 22	S 42, 46, 52, 56, 63	5650	−	E	
261*a*	30	11 1 —11 22	S 42, 44, 52, 57, 62	5650	−	E	
261*b*	30	11 24 —11 45	S 39, 44, 52, 57, 62, 68	5650	+	E	
264*a*	31	8 33 — 9 19	S 47, 51, 60, 66, 73, 79	5650	+	E	
264*b*	31	9 20 —10 3	S 47, 51, 60, 66, 73, 79	5650	−	E	
265*a*	31	10 8 —11 0	S 48, 51, 59, 65, 72, 82	5650	−	E	
265*b*	31	11 1 —11 54	S 47, 51, 59, 65, 72, 82	5650	+	E	
271*b*	Feb. 9	10 54 —11 19	N 64, 58, 47, 42, 37, 35	5650	+	E	
272*a*	10	8 43 — 9 23	N 68, 58, 47, 41, 36	5650	+	E	
273*b*	10	10 50 —11 29	N 68, 60, 50, 44, 40, 36	5650	+	E	
274*a*	10	12 40 — 1 26	N 70, 63, 50, 45, 40, 36	5650	+	W	
275*a*	10	2 20 — 2 41	N 27, 20, 16, 12	5650	−	W	
275*b*	10	2 42 — 3 4	N 30, 26, 20, 16, 12, 9	5650	+		
276*a*	11	8 30 — 9 12	N 71, 59, 49, 44, 38	5650	+	E	

TABLE III—*Continued*

T′ 279*b*.....	Feb. 13	11 10 —11 59	S 47, 51, 59, 65, 72, 78	5650	+	E	
280*a*.....	13	1 25 — 1 45	S 24, 28, 34, 37	5650	+	W	
283*a*.....	14	2 53 — 3 17	S 19, 22, 28, 31, 35, 39	6300	+	W	λ/4 for λ 6300 has
283*b*.....	14	3 18 — 4 00	S 37, 41, 48, 51, 60, 64	6300	+	W	been remounted
288*a*.....	18	9 10 — 9 23	N 21, 17, 14, 10, 5, 1	6300	+	E	
288*b*.....	18	9 36 — 9 57	N 21, 17, 14, 10, 5, 1	6300	+	E	
290*a*.....	18	10 55 —11 15	S 0, 3, 7, 12, 17, 21	6300	−	E	
290*b*.....	18	11 15 —11 35	S 0, 3, 7, 12, 17, 21	6300	−	E	

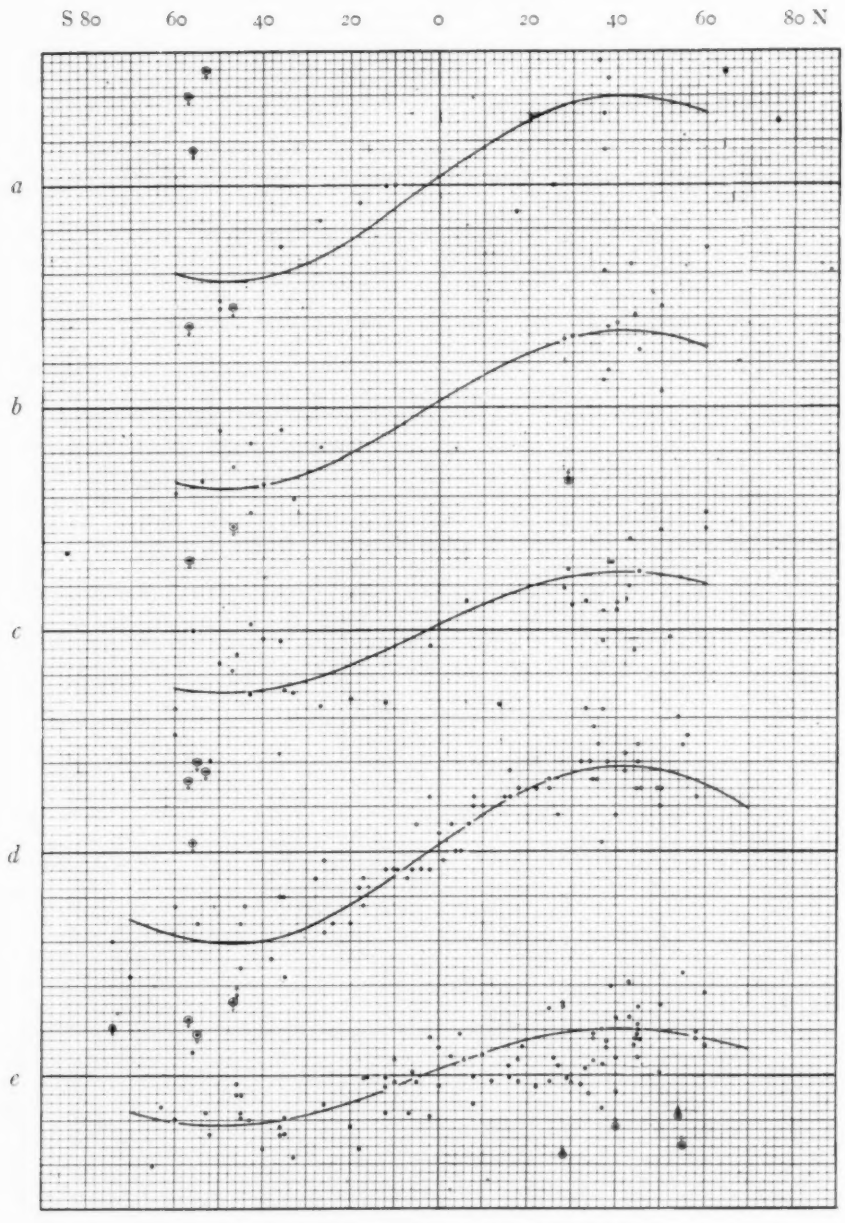


FIG. 2.—Displacements for the first series

a, Mean displacements for λλ 5812 and 5828 measured by Miss Lasby; *b*, λ 5812 measured by Mr. Van Maanen; *c*, λ 5828 measured by Mr. Van Maanen; *d*, λ 5930 measured by Miss Lasby; *e*, λ 5930 measured by Mr. Van Maanen.

Vertical scale: 1 division = 0.001 mm.

lines, which show no shifts greater than the errors of setting. The work of measuring the three lines having the largest displacements on Plates 3 and 4 was accordingly continued by Miss Lasby, with the results collected in Tables IV and VI, which give the values of the relative displacements, expressed in thousandths of a millimeter and arranged in order of decreasing latitude. In general, as is indicated by the numerical values of the displacements and by Fig. 2, *a* and *d*, Miss Lasby's measures show positive displacements, gradually decreasing from a maximum of about 0.010 mm (0.002 Ångström) for points in middle northern latitudes (N. 45°)

TABLE IV

FIRST SERIES—λλ 5812.139 AND 5828.097 IN THIRD-ORDER SPECTRUM
Measures by Miss Lasby

LAT.	Δ		MEAN Δ	PLATE NO.
	λ 5812	λ 5828		
N 38°	+12	+12	+12	25
37	+ 8	+ 8	+ 8	29
37	+ 6	+ 3	+ 4	30
36	+13	+14	+14	23
N 33	+17	+16	+16	24
S 10	− 1	+ 2	0	48
12	− 1	0	0	19
18	− 2	− 1	− 2	48
27	− 4	− 4	− 4	48
36	− 6	− 8	− 7	48
50	−13	−14	−14	4
53	(+12)	(+14)	(+13)	18
56	(+ 4)	(+ 4)	(+ 4)	27
S 57	(+ 8)	(+12)	(+10)	26

to zero near the equator, and increasing again, with reversed sign, toward a similar maximum in middle southern latitudes (±S. 45°). There are a few discrepancies in sign, but most of the measures are remarkably consistent. Every precaution was taken to prevent bias on the part of the observer, who did not know the latitude or hemisphere of any plate.

But in spite of the excellent internal agreement of the measures, the fact that the lines in Table I having the largest values of $\Delta\lambda/\lambda^2$ showed no appreciable displacement raised a question as to the possibility that the observed shifts of λ 5812, λ 5828, and λ 5930 might be due to some obscure source of error. This could hardly lie in the measures themselves, as Miss Lasby had acquired much

experience in the measurement of photographs of spectra showing the solar rotation, and in other investigations of an equally exacting nature. The high precision attained in this work of Mr. Adams and Miss Lasby is shown by the published results,[1] which have been confirmed by the interferometer measures of Perot. In the present case a number of check measures were kindly made by Mr. Adams.

TABLE V

FIRST SERIES—λλ 5812.139 AND 5828.097 IN THIRD-ORDER SPECTRUM
Measures by Mr. Van Maanen

LAT.	Δ λ 5812	Δ λ 5828	PLATE NO.	LAT.	Δ λ 5812	Δ λ 5828	PLATE NO.
N 60°	.....	+13.2	25	N 6°	.....	+ 3.3	21
60	+12.9	+11.4	23	S 2	.....	− 1.8	21
52	.....	− 0.8	10	12	.....	− 8.2	21
50	+ 1.7	.....	39	20	.....	− 7.9	21
50	+11.2	+11.2	37	27	− 4.5	− 8.6	48
45	+ 6.4	+ 6.6	24	33	−10.2	− 7.1	41
44	+10.2	− 2.3	29	35	.....	− 6.9	21
43	+16.0	+ 5.0	23	36	− 2.5	− 1.3	48
43	.....	+10.2	25	40	− 8.6	− 1.0	11
42	+ 0.3*	+ 1.7*	31	43	−11.9	+ 0.7	4
40	+ 9.4	+ 2.3	10	43	− 4.0	− 7.3	41
40	.....	+ 3.0	37	46	.....	− 2.6	27
38	+ 4.1	+ 7.6	39	47	− 6.6	− 4.6	48
38	+ 8.9	+ 7.6	24	47	(+11.2)	(+11.6)	26
37	+15.2	+ 2.1	30	50	− 2.6	− 3.6	4
37	+ 3.0	− 1.2	29	52	− 8.2	−14.8	11
33	.....	+ 3.3	25	56	.....	0.0	27
32	0.0*	.....	31	57	(+ 9.2)	(+ 7.9)	26
30	+ 7.9	+ 2.8	30	60	.....	−11.9	4
29	(− 8.2)	+ 6.9	29	S 60	− 9.6	− 8.9	48
N 28	+ 7.6	+ 4.6	24				

*Half-wave plate −22°.5. Displacements should be zero.

Most of these agree with Miss Lasby's measures in sign, but there appears to be a systematic tendency on her part toward larger values. Several other members of the Observatory staff also measured a number of the plates, but although in many cases the internal agreement of their measures was good, the results of different observers for the same plate frequently differed widely. It appeared that a radically different method of measurement, which would

[1] Adams and Lasby, "An Investigation of the Rotation Period of the Sun," *Publications of the Carnegie Institution of Washington*, No. 138.

TABLE VI

FIRST SERIES—λ 5929.898 IN THIRD-ORDER SPECTRUM
Measures by Miss Lasby and Mr. Van Maanen

LAT.	Δ L.	Δ V. M.	PLATE NO.	LAT.	Δ L.	Δ V. M.	PLATE NO.
N 60°	....	+ 3.3	23	N 26°	....	+ 1.8	42
60	....	+ 9.2	25	25	+ 7	− 0.7	44
58	+ 6	+ 4.0	42	25	+ 8	+ 7.4	55
58	....	+ 4.8	55	22	+ 7	− 1.3	46
56	+13	.....	10	19	....	+ 3.1	51
55	....	(− 7.9)	40	18	....	− 0.7	42
55	+12	11.4	44	18	+ 7	+ 1.7	49
54	+15	(− 4.5)	46	16	+ 6	+ 1.0	44
50	+ 7	.....	36	16	+ 9	− 0.2	55
50	+ 5	+ 7.9	37	15	+ 6	.....	46
50	+ 7	− 0.3	39	12	....	− 0.7	51
45	+ 7	+ 4.0	35	10	+ 5	+ 2.3	49
45	....	+ 4.5	24	8	+ 5	− 3.3	44
45	+ 7	+ 4.0	40	8	+ 6	− 0.2	55
45	+10	+ 5.1	42	7	+ 3	.....	46
45	+12	+ 2.0	44	6	....	+ 1.6	21
45	....	+ 7.6	49	5	0	+ 4.6	51
45	....	+ 5.4	55	4	0	.....	54
44	....	+ 3.3	29	3	+ 3	+ 2.0	49
43	....	+10.4	25	N 1	− 1	.....	55
43	....	+ 6.4	23	0	....	+ 3.0	15
42	+ 9	.....	32	0	+ 2	− 1.3	44
42	+ 3*	+ 4.0*	33	S 2	....	− 4.6	21
42	+11	+ 4.0	46	2	+ 6	.....	43
40	+ 4	+ 6.3	34	2	− 2	+ 4.1	45
40	....	+ 2.8	37	4	− 2	.....	54
40	....	− 1.8	17	5	+ 3	− 0.7	49
40	....	(− 5.6)	10	6	− 2	+ 0.3	15
39	....	+ 9.9	40	7	....	− 4.3	19
38	....	+ 3.0	24	7	− 3	.....	44
38	+10	.....	25	10	− 2	− 0.8	45
38	....	+ 3.6	39	10	− 2	+ 1.7	50
37	....	− 3.6	29	12	....	− 1.3	21
37	+ 1	+ 1.1	30	12	....	− 0.3	15
37	+16	+ 5.0	49	12	....	− 4.3	19
36	+12	.....	23	12	− 2	.....	54
35	+14	+ 4.1	42	16	− 4	.....	47
35	+ 8	+ 1.5	44	17	− 6	− 0.3	45
35	+ 8	+ 4.5	55	17	− 3	− 0.2	50
34	+10	− 2.0	17	18	....	− 8.4	19
33	+16	.....	24	18	− 4	.....	43
33	....	+ 0.7	25	20	....	− 5.6	21
32	+10	− 1.0	46	20	− 8	.....	54
30	....	− 0.8	23	24	− 8	.....	47
30	....	0.0	30	26	− 9	− 3.3	45
29	....	− 0.3	29	26	....	− 1.0	50
28	....	(− 8.9)	17	28	− 3	.....	43
28	....	+ 7.6	24	33	....	− 9.2	41
28	....	+ 7.8	51	35	....	− 6.6	21
N 27	+ 4	+ 1.0	49	S 35	−14	− 4.7	45

TABLE VI—*Continued*

Lat.	Δ		Plate No	Lat.	Δ		Plate No.
	L.	V. M.			L.	V. M.	
S 36°....	− 5	− 5.9	43	S 47°....	....	(+ 8.2)	26
36.....	− 5	.....	47	52.....	....	− 6.6	11
36.....	....	− 6.6	50	53.....	(+ 9)	− 4.1	18
38.....	−12	.....	41	55.....	(+10)	(+ 4.6)	14
40.....	....	− 8.4	11	55.....	....	− 3.6*	28
43.....	....	− 5.0	41	55.....	− 8	.....	47
44.....	− 6	.....	47	56.....	(+ 1)	+ 2.5	27
45.....	....	− 2.3	18	57.....	(+ 8)	(+ 6.2)	26
45.....	− 8	− 4.3	43	60.....	− 6	− 5.0	43
45.....	−13	.....	45	63.....	....	− 3.5	18
45.....	....	− 4.6	50	65.....	....	−10.2	14
46.....	−16	.....	11	70.....	−14	.....	43
46.....	....	− 1.0	14	74.....	....	(+ 5.3)	26
46.....	....	+ 7.9*	28	S 74.....	−10	.....	47
S 46.....	....	− 2.1	27				

* Half-wave plate at −22°.5. Displacements should be zero.

tend to eliminate or at least to modify possible systematic errors, would be of great service as a check on the results.

This was provided in the form of a parallel plate micrometer, the use of which was suggested by Mr. Pease. A plate of plane, parallel glass 12 mm long, 2 mm wide, and 1.14 mm thick, mounted so as to rotate about an axis lying centrally in the plane of the plate, at right angles to its longest dimension, is supported just above the negative to be measured. Any given strip of the spectrum, corresponding to a section of the quarter-wave plate, can be brought under the parallel plate, which exactly equals it in width. The spectrum is observed through an achromatic doublet, magnifying about three diameters. By inclining the parallel plate, the line seen through it is displaced until it comes into coincidence with the line of an adjoining strip. The angle through which the plate is turned is given by an index moving over a divided arc.[1]

The coincidence reading having been obtained, an adjacent spectrum strip is brought underneath the micrometer plate by

[1] In a perfected form of this micrometer, the parallel plate is mounted in the focal plane of the positive eyepiece of a compound microscope. To preserve the optical symmetry of the system, plates of plane parallel glass are fixed on either side of the movable plate, with their surfaces parallel to the plane of the negative. For use with low powers, however, the simple instrument described above is very satisfactory.

shifting the negative. The difference between the coincidence reading for this position and that first obtained gives the angular rotation of the plate corresponding to twice the relative displacement of the spectrum lines. Since the rotation is always small, the multiplication of the difference by a constant gives the value of the displacement expressed in thousandths of a millimeter. Three coincidence readings are made for each spectrum strip and usually six successive strips, involving four settings of the plate, are measured in both directions, the results being combined to a mean value corresponding to the mean latitude of the strips.

Immediately after the construction of the parallel plate micrometer, extended sets of measures of the plates of the first series were undertaken by two members of the Computing Division. Here, again, successive settings upon the same line usually showed a high degree of accordance, but when the displacements for different latitudes were compared, there was no definite agreement or progression in the values when arranged according to latitude. The same was true of a preliminary set of measures made by Mr. Van Maanen with the same instrument. The systematic errors for a latitude (accidental for the series) were so large as completely to mask the displacements revealed by the later measures. None of the three observers had had previous experience with a micrometer of this form, and the result is not surprising when the character of the lines is considered. A continuation of the measures by Mr. Van Maanen led to different results, however. A remeasurement of the plates of the first series showed a decrease in the accidental errors, and brought to light a variation of the displacement with the latitude similar to that shown by the measures of Miss Lasby. It should be repeated that, throughout all the measures, the greatest care was taken to avoid the unconscious effect produced by a knowledge of the conditions under which the photographs were made. As already stated, the measurer knew neither the hemisphere, the latitude, nor the progression of latitudes on any plate; for, after the latitudes of the regions to be measured had been determined, the plate was cut into strips to which arbitrary letters or figures were assigned, by an individual other than the one who made the measures. For the later series

there was a further uncertainty on the part of the measurer, owing to the fact that the quarter-wave plate was used in both the normal and inverted positions (+ and −). The two sections of a photograph were marked *a* and *b* by the observer, usually Mr. Ellerman or Mr. Kohlschütter, with a corresponding entry in the record book for subsequent identification. Sometimes *a* referred to the + position and sometimes to the −; and not infrequently both the *a* and *b* sections were made with the quarter-wave plate in the same position. The matter is explained in detail, because this method of procedure, together with the general consistency of the results included in the discussion, is the basis of the rejection of all of the measures of the first series except those by Miss Lasby and the later set by Mr. Van Maanen. His results are given in Tables V and VI and are shown graphically in Fig. 2, *b*, *c*, and *e*. Mr. Van Maanen's measures agree, on the average, in showing positive displacements in the northern hemisphere and negative ones in the southern, with values decreasing toward a minimum near the equator, but the average displacement is much smaller than that found by Miss Lasby. The systematic differences between the two observers may be due to the very different methods of measurement employed.

SECOND SERIES

As already explained, the observations of this series were made in the second-order spectrum with the 75-foot spectrograph. The difficulties experienced in the measurement of the broad and diffuse lines of the third-order plates of the first series suggested the possibility that the increased contrast and sharpness of the second order might more than offset the disadvantage of the smaller scale, which with the grating employed is 1 Ångström = 2.95 mm at λ 5900. As the scale for the third order is 1 Ångström = 4.90 mm, the dispersion is 1.66 times that of the second order. The measures of the second-order plates were all made by Mr. Van Maanen with the parallel plate micrometer. The results, reduced to the scale of the third order, are collected in Table VII, and are illustrated in Fig. 3. The regular use of the quarter-wave plate in both the normal and the inverted positions began with this series. The

TABLE VII

SECOND SERIES—λλ 5812.139, 5828.097, AND 5929.898 IN SECOND-ORDER SPECTRUM. DISPLACEMENTS REDUCED TO SCALE OF THIRD ORDER

Measures by Mr. Van Maanen

LAT.	Δ						PLATE NO.
	λ 5812		λ 5828		λ 5930		
	+	−	+	−	+	−	
N 63°	.....	.....	.....	.....	−3.3	+2.8	143
62	+1.1	−1.1	−2.8	−0.6	0.0	−1.6	137
60	.....	.....	.....	.....	+3.3	−1.9	125
60	.....	.....	.....	.....	−2.8	−0.6	126
60	+6.3	+1.1	+6.0	−1.6	−6.9	−1.6	154
58	.....	.....	.....	.....	+0.3	.....	127
58	.....	.....	.....	.....	+2.2	.....	127
58	.....	.....	.....	.....	−3.3	+1.1	129
56	(−9.9)	+3.3	(−10.4)	+3.0	+2.5	+5.5	130
55	+6.6	−9.4	+3.8	(−15.4)	+2.2	−3.0	131
53	.....	.....	.....	.....	−1.9	+1.6	149
43	+7.2	−5.5	+3.3	−7.7	+0.3	+1.9	146
43	.....	−5.0	.....	−11.0	.....	+5.5	149
42	+7.7	.....	+0.6	.....	−7.2	.....	149
38	.....	.....	.....	.....	+3.0	.....	127
38	.....	.....	.....	.....	−0.6	.....	127
38	.....	.....	.....	.....	−1.4	+1.1	129
38	(−7.2)	(+6.9)	+3.8	(+6.6)	−4.1	+1.1	130
38	.....	.....	.....	.....	−1.9	+1.9	143
37	.....	.....	.....	.....	−3.0	−1.6	125
37	.....	.....	.....	.....	−1.1	+2.2	126
36	+5.0	−1.6	+6.0	−2.2	+1.6	−0.0	137
35	+5.8	+2.8	+2.8	0.0	−4.4	−3.6	154
34	0.0	+5.0	+3.3	(+10.4)	+5.0	−2.2	131
11	+4.1	−2.2	−2.5	−0.6	−3.6	+4.4	155
1	.....	−3.6	+2.8	−2.2	0.0	0.0	148
10	+3.8	+4.7	+1.9	−3.3	+1.6	−4.1	153
N 7	.....	.....	.....	.....	−5.5	−6.0	159
S 2	−8.2	+2.8	+7.4	+8.2	−6.3	−0.8	145
4	.....	.....	.....	.....	−6.0	−2.2	159
4	+3.8	+7.7	−1.1	−0.6	−2.2	+7.7	155
5	+2.2	−1.1	+0.8	−8.0	−0.8	0.0	148
5	.....	−2.8	.....	.....	+4.4	−5.0	153
31	.....	.....	.....	.....	+3.3	−7.2	147
31	.....	.....	.....	.....	0.0	−2.2	156
32	−6.0	+8.0	−7.2	+1.1	−8.2	+4.4	135
34	.....	.....	.....	.....	−1.6	+3.8	133
34	.....	.....	.....	.....	−1.6	.....	134
34	.....	.....	.....	.....	0.0	.....	134
35	−6.0	+1.4	−3.3	+1.6	+6.3	−1.1	142
41	.....	.....	.....	.....	−6.6	+1.9	144
41	+0.6	+5.5	−6.3	+3.8	−6.0	+4.4	160
51	.....	.....	.....	.....	+3.3	−6.0	156
53	−3.3	+1.1	+1.1	+9.9	−3.8	+5.5	135
53	.....	.....	.....	.....	+2.8	−5.5	147
55	.....	.....	.....	.....	−5.5	+7.7	133
55	.....	.....	.....	.....	+1.6	.....	134
55	.....	.....	.....	.....	+0.3	.....	134
60	.....	.....	.....	.....	−2.2	+0.8	144
S 62	0.0	+10.4	−3.3	+7.7	+3.8	+1.1	142

results are given separately and are distinguished by the signs + (normal) and − (inverted) in the table heading. The preponderance of the number of cases of opposite sign in the + and − columns is at once evident. As was anticipated, the measurement of the plates was less fatiguing than in the case of the third order, but

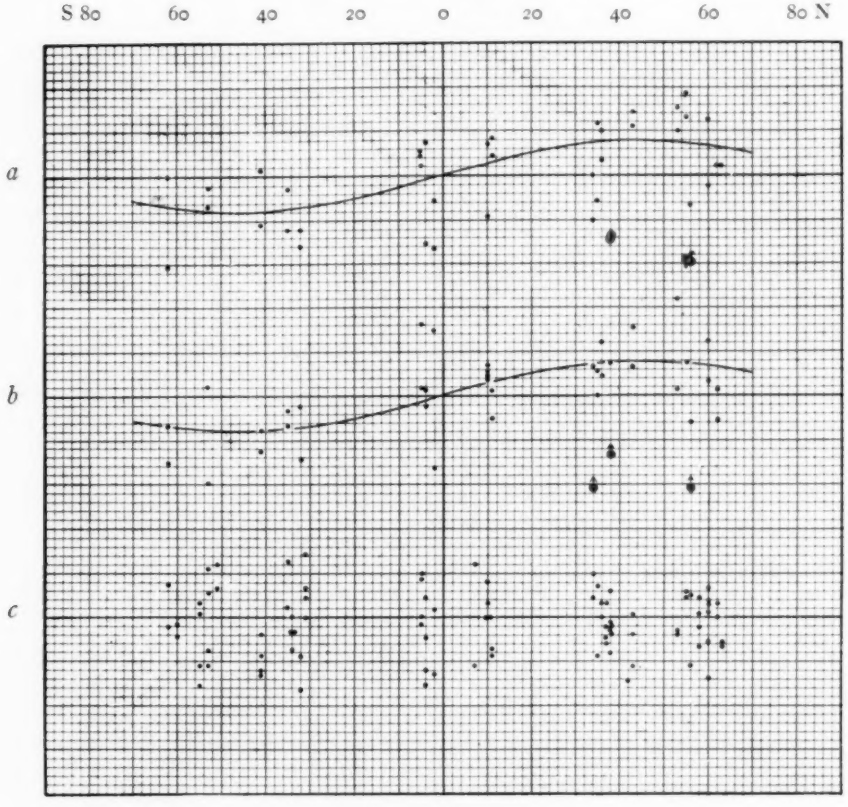


FIG. 3.—Displacements for the second series, measured by Mr. Van Maanen *a*, λ 5812; *b*, λ 5828; *c*, λ 5930. Vertical scale: 1 division = 0.001 mm.

the results did not confirm those of the first series. They raise, however, a question of considerable interest which is discussed below (p. 74).

As stated on p. 42, a number of second-order plates were also made with the small solar image formed by the 30-foot (9.1 m)

objective. None of the measures on these plates is included in the discussion. They give negative results which are not printed in detail, as the difficulty experienced with the second-order plates is sufficiently indicated by the measures on λ 5930 in the second series.

THIRD AND FOURTH SERIES

The negative character of the results from the observations in the second-order spectrum led to a return to the third order for the third and fourth series. The observing arrangements were the same for these two series, and the distinction between them is only that of the lines observed and the dates covered by the observations. It will be noted, however, that the fourth series includes mainly the higher latitudes, both N. and S.

The results of Mr. Van Maanen's measures with the parallel plate micrometer for these two series are given in Tables VIII and IX, the unit, as before, being 0.001 mm. An examination of the tables, or a glance at Figs. 4–6, shows at once a variation of the displacement with the latitude similar to that revealed by the first series. The detailed discussion of all of the results is given below (p. 73).

SOURCES OF ERROR

The systematic difference between the measures made by Miss Lasby with the Toepfer comparator and those by Mr. Van Maanen with the parallel plate micrometer, referred to in connection with the first series, made it desirable to provide a method of determining the nature and magnitude of systematic errors incident to the measurement of any type of spectral line. An instrument for this purpose would be of great service in giving the personal equation of each observer taking part in an investigation. Apparently the best means of accomplishing this is by the measurement of a known displacement of the line in question. Such a displacement is easily produced by means of a plane parallel glass plate, mounted before the negative, and inclined at a known angle. The calculated displacement can be checked, if desired, by the measurement under a high-power microscope of a fine line engraved on the film of the negative parallel to the lines of the spectrum.

TABLE VIII

THIRD SERIES—λ 5929.898 IN THIRD-ORDER SPECTRUM
Measures by Mr. Van Maanen

LAT.	Δ		PLATE NO.	LAT.	Δ		PLATE NO.
	+	−			+	−	
N 66°....	+ 1.6	− 5.8	230	N 35.....	+ 6.4	.....	200
62.....	+ 6.8	.....	181	35.....	+ 3.6	.....	233
60.....	+ 5.3	.....	193	32.....	.....	− 1.3	200
60.....	+ 9.9	− 3.0	184	11.....	− 7.9	− 3.0	203
59.....	− 3.3	.....	212	7.....	+ 6.3	− 4.4	203
59.....	.....	+ 4.0	200	N 2.....	− 5.0	+ 0.7	203
59.....	+ 7.6	.....	223	S 2.....	− 0.1	− 1.6	203
59.....	+ 3.3	− 4.3	230	6.....	− 0.3	+ 1.3	203
58.....	+ 2.8	.....	200	33.....	.....	+ 8.9	187
56.....	+ 9.2	.....	233	33.....	+ 5.0	+10.9	220
54.....	− 0.7	.....	212	33.....	+ 4.6	.....	212
54.....	+ 7.1	.....	223	33.....	.....	− 5.0	211
54.....	+ 5.6	.....	223	33.....	− 8.6	+ 4.3	196
53.....	+ 5.1	.....	200	34.....	− 9.7	(+19.5)	185
53.....	+ 1.6	− 4.8	230	36.....	− 9.9	.....	194
53.....	+11.9	− 7.8	184	37.....	+ 4.8	.....	233
53.....	+ 6.9	.....	181	37.....	− 2.6	+ 5.6	232
52.....	.....	− 1.2	200	37.....	(+ 5.6)	.....	212
52.....	+ 6.9	.....	233	37.....	.....	(−5.9)	211
51.....	+ 4.6	− 1.6	217	38.....	− 5.3	+14.7	185
49.....	+ 4.0	− 5.9	230	38.....	− 6.6	+ 6.9	226
48.....	0.0	.....	200	38.....	.....	+ 9.2	220
47.....	+ 6.3	− 5.4	217	38.....	− 0.7	.....	196
46.....	− 4.1	.....	212	39.....	− 5.6	.....	233
46.....	+ 6.6	− 2.5	181	39.....	− 7.6	+ 4.3	232
45.....	+ 9.2	.....	223	42.....	− 3.1	.....	194
45.....	+ 5.0	.....	223	43.....	(+ 8.6)	.....	212
45.....	− 2.0	− 1.2	230	43.....	.....	− 2.0	211
45.....	+ 6.3	−11.6	184	45.....	− 9.9	+ 2.0	226
43.....	+ 6.3	−11.4	193	45.....	(+ 7.4)	+ 8.1	220
43.....	+ 4.1	(+ 8.6)	200	47.....	−15.5	+14.4	185
43.....	+ 6.9	.....	233	47.....	− 2.6	.....	233
43.....	.....	− 6.9	192	47.....	− 5.3	+ 7.6	232
42.....	(− 6.3)	.....	212	47.....	+ 5.0	.....	212
42.....	+ 5.3	− 9.6	181	47.....	− 8.6	+12.5	196
41.....	.....	+ 3.5	200	48.....	.....	+ 1.3	211
41.....	+ 6.9	.....	223	49.....	− 7.3	+ 5.8	226
41.....	+ 3.3	.....	223	50.....	+ 5.0	+11.6	220
41.....	.....	− 4.3	230	52.....	−10.6	+12.2	185
39.....	+ 6.3	−11.4	193	52.....	− 3.5	.....	233
39.....	+ 2.3	.....	230	52.....	− 6.9	+ 5.0	232
39.....	+ 6.6	.....	233	52.....	− 6.3	+ 9.6	196
39.....	.....	− 4.6	192	53.....	+ 3.3	+ 4.3	220
37.....	(− 8.9)	.....	212	53.....	(+ 6.9)	.....	212
37.....	.....	− 4.4	181	55.....	− 4.4	+ 1.0	226
36.....	.....	+ 2.6	200	57.....	− 5.6	.....	233
36.....	+ 3.1	.....	223	S 57.....	− 5.0	+ 6.4	232
N 36.....	+ 5.8	.....	223				

TABLE IX

FOURTH SERIES—λλ 5812.139 AND 5828.097 IN THIRD-ORDER SPECTRUM
Measures by Mr. Van Maanen

LAT.	Δ				PLATE NO.
	λ 5812		λ 5828		
	+	−	+	−	
N 71°	−2.6	.....	+2.6	.....	276
70	+0.3	.....	+0.7	.....	274
68	+0.7	.....	+1.6	.....	272
68	+4.4	.....	+2.0	.....	273
64	+1.3	.....	+1.8	.....	271
63	+4.0	.....	+1.8	.....	274
60	+4.8	.....	+3.0	.....	273
59	+1.8	.....	+2.3	.....	276
58	+3.6	.....	+0.8	.....	272
58	+2.6	.....	+3.3	.....	271
50	+4.1	.....	+3.6	.....	273
50	+5.0	.....	+3.0	.....	274
49	+4.3	.....	+6.9	.....	276
47	+8.2	.....	+5.3	.....	272
47	(−1.6)	.....	+6.3	.....	271
45	+4.6	.....	+7.4	.....	274
44	+6.9	.....	(+9.6)	.....	276
44	+5.9	.....	+5.0	.....	273
42	+6.9	.....	+6.9	.....	271
41	+5.3	.....	+5.3	.....	272
40	+4.6	.....	+5.0	.....	273
40	+8.2	.....	+2.6	.....	274
38	+4.0	.....	+5.0	.....	276
37	+5.3	.....	+2.8	.....	271
36	+2.3	.....	+1.0	.....	272
36	+5.0	.....	+5.6	.....	273
36	+5.0	.....	+7.3	.....	274
35	+3.0	.....	+4.3	.....	271
30	+5.3	.....	+3.3	.....	275
27	.....	(+5.9)	.....	(+4.0)	275
26	+2.6	.....	+3.3	.....	275
21	+3.0	.....	+3.0	.....	288
21	+6.0	.....	+4.4	.....	288
20	+3.5	−3.0	+5.3	−3.0	275
17	+5.3	.....	+4.6	.....	288
17	+2.5	.....	+3.5	.....	288
16	+2.5	−2.5	+2.3	−3.6	275
14	+3.3	.....	−0.5	.....	288
14	−0.3	.....	+0.3	.....	288
12	+4.4	−2.0	+2.0	−1.8	275
10	+1.5	.....	+0.8	.....	288
10	+2.5	.....	+0.5	.....	288
9	+2.6	.....	0.0	.....	275
5	+0.3	.....	+1.2	.....	288
5	+2.0	.....	+0.8	.....	288
1	+2.0	.....	−0.3	.....	288

TABLE IX—*Continued*

Lat.	Δ λ 5812 +	Δ λ 5812 −	Δ λ 5828 +	Δ λ 5828 −	Plate No.
N 1°	−2.6	.....	−1.5	.....	288
0	.....	−1.3	.....	−0.7	290
0	.....	−1.3	.....	+0.7	290
S 3	.....	+1.6	.....	0.0	290
3	.....	+1.5	.....	+0.3	290
7	.....	+2.0	.....	+1.0	290
7	.....	−1.3	.....	.....	290
12	.....	+3.6	.....	+4.0	290
12	.....	−0.5	.....	+1.8	290
17	.....	+4.6	.....	+2.6	290
17	.....	+4.1	.....	+2.8	290
19	−1.6	.....	−2.6	.....	283
21	.....	+5.6	.....	+2.3	290
21	.....	+2.3	.....	+3.0	290
22	−2.6	.....	−2.0	.....	283
24	(+7.9)	.....	(+7.4)	.....	280
28	−2.6	.....	−4.0	.....	283
28	(+4.8)	.....	(+5.9)	.....	280
31	−2.6	.....	−5.3	.....	283
34	+1.0	.....	(+3.3)	.....	280
35	−4.3	.....	−5.9	.....	283
37	−5.6	.....	−2.6	.....	283
37	.....	.....	(+4.6)	.....	280
39	−4.6	.....	−1.3	.....	261
39	−1.3	.....	−5.3	.....	283
41	−4.6	.....	−3.6	.....	283
42	.....	+7.9	.....	+5.1	261
42	−3.0	+4.4	−4.3	+4.6	258
44	−8.2	+4.3	−4.0	+3.5	261
46	(+3.0)	+5.3	(+5.6)	+3.6	258
47	(+8.2)	.....	(+5.6)	.....	279
47	(+1.6)	.....	−3.0	.....	265
47	−3.0	+6.9	−4.3	+2.6	264
47	−4.6	.....	−3.0	.....	255
48	.....	(−4.0)	.....	+6.6	265
48	−4.6	.....	−2.6	.....	283
49	.....	+3.6	.....	+4.3	255
51	(+4.3)	.....	(+2.0)	.....	279
51	−1.6	.....	−2.0	.....	283
51	−9.6	+8.7	−5.9	+4.3	265
51	−2.3	+4.6	−2.6	+5.0	264
52	−4.0	+5.0	−2.0	+6.4	255
52	−7.6	+4.3	−0.8	+3.3	261
52	−2.6	+5.3	−5.9	+4.1	258
56	−5.0	+3.0	.....	+2.6	258
57	−5.0	+7.4	−5.0	+4.0	261
59	−4.6	+5.9	−1.3	+2.6	265
59	(+4.6)	.....	(+2.8)	.....	279
60	−2.0	.....	−2.3	.....	283
60	+0.3	+1.8	−4.1	+4.0	264

TABLE IX—*Continued*

Lat	Δ				Plate No.
	λ 5812		λ 5828		
	+	−	+	−	
S 60°	−4.0	+4.3	−3.0	+3.6	255
62	−3.0	+0.8	−7.4	+8.1	261
63	−4.6	+4.3	−2.0	−0.5	258
64	−1.3	.....	−0.5	.....	283
65	0.0	.....	−1.6	+1.6	265
65	.....	.....	(+3.6)	.....	279
66	−1.5	+0.3	−3.0	+4.3	255
66	−0.7	+0.3	−0.3	+2.0	264
68	−2.6	.....	−1.3	.....	261
72	(+2.0)	.....	+1.3	.....	279
72	−0.7	+4.6	−0.8	+0.7	265
72	−1.0	.....	−1.3	.....	255
73	.....	+0.3	.....	+0.8	255
73	+0.2	+1.3	−1.6	+0.8	264
78	+2.3	.....	+2.1	.....	279
79	+1.0	.....	−0.3	.....	255
79	(+3.0)	−0.2	−0.3	+0.3	264
S 82	+3.3	+1.3	−0.8	+1.0	265

A preliminary test of this method had indicated that under certain conditions, which probably depend upon the granular structure of the plate,[1] a line actually displaced toward the red may appear to be displaced toward the violet. Mr. Babcock, to whom I am indebted for much assistance in connection with the present work, selected a third-order photograph at λ 5930, taken at the sun's equator, and showing no relative displacement of the line in successive strips. A plane parallel glass plate, equal in width to one of the strips, was mounted above it at a small angle, sufficient to displace a line seen through it by +0.004 mm. Two photographs were taken of the spectrum, showing the displaced line and two adjoining strips, which were covered with plates of plane parallel glass, with their faces parallel to the film, to compensate for focus. These photographs, which were exactly alike in all known respects, may be called A and A′. Another photograph (B), show-

[1] Perrine, "Some Results of a Study of the Grain and Structure of Photographic Films," *Lick Observatory Bulletin*, No. 143; "Results of Some Further Studies of the Structure of Photographic Films and the Effect on Measures of Star Images," *ibid.*, No. 148.

ing a displacement of −0.025 mm, was also taken. The line λ 5930 was then measured on the three photographs by Mr. Adams and Mr. Van Maanen, after Mr. Babcock had determined the actual displacements by measuring with a high power several sharp artificial lines on both sides of λ 5930. The results of the measures of λ 5930 made by Mr. Adams with a measuring machine of the ordinary kind (micrometer screw and cross-wire), and by Mr. Van Maanen with the parallel plate micrometer previously described, are given below.

Plate	Actual Displacement	Measured Displacement	
		Adams	Van Maanen
A	+0.004 mm	+0.002 mm	+0.0036 mm
A′	+0.004	−0.001	−0.004
B	−0.025	−0.026	−0.014

It thus appears that two apparently identical copies of the same photograph may differ in such a way as to transform an actual positive displacement into an apparent negative one.

The method appears so promising that a special instrument is being constructed for the further study of personal equation and systematic errors. In this instrument a strip of plane parallel glass, mounted between two fixed strips of equal width (2 mm) can be set at any desired angle with the plane of the negative. The resulting displacement of the solar line is then measured with an ordinary eyepiece micrometer or with a parallel plate micrometer easily substituted for it.

So much for systematic errors, the further discussion of which must await the completion of this instrument. As for errors due to other causes, we have already seen, in the case of the iodine absorption spectrum, the absence of such disturbing effects as might enter from lack of stability, changes of temperature, imperfect resolution, or small linear dispersion in the spectrograph. Unfortunately the solar lines are much less sharp than those of iodine, and the difficulties of measurement are far greater. This sets a limit to the accuracy attainable, but the errors arising from this source are of an accidental nature, and certainly could not be

conceived to account for the observed relationship between latitude and displacement, or the reversal of sign in the two hemispheres. Other possible sources of error are: (1) unequal illumination of the grating; (2) heating of the slit jaws; (3) imperfect orientation, centering, and guiding, and poor definition or distortion of the solar image; (4) elliptical polarization by the silvered coelostat mirrors; (5) polarization in the spectrograph; (6) imperfect correction of the quarter-wave plate.

These may be discussed in turn:

1. As a complete set of measures involves the use of four quarter-wave strips, covering 8 mm of the slit, and as the light from each part of the slit falls upon the grating in a slightly different way, very small differential displacements might conceivably be produced. It has been shown above, however, that the ruled surface in the first series of observations was much more than covered by the beam from the objective (ratio of aperture to focal length = 1/60, while the corresponding ratio for the spectrograph = 1/181), and in the later work with the objective of 150 feet (45.7 m) focal length there was still a safe margin. A decisive test for such displacements is afforded by the atmospheric lines, one of which falls very near λ 5930. This was measured by Miss Lasby on a large number of plates, but never showed a displacement exceeding 0.002 mm, which is well within the errors of measurement.

2. With a ratio of aperture to focal length of 1/60 in the telescope, the heating of the slit jaws is very slight, especially after the introduction of the quarter-wave plate and Nicol. In any event, this would produce merely a slight widening of the slit, without effect in purely differential measures. A sufficient check is again afforded by the atmospheric lines.

3. On account of the solar rotation, displacements of a solar line may result from errors in the orientation or centering of the solar image, or from imperfect guiding during the exposure. Poor definition of the image, due to atmospheric disturbance, change of focus or astigmatism of the mirrors, may introduce a small effect of the same kind. For the 17-cm solar image, a departure of the slit from the central meridian of the sun amounting to 1.5 mm may be regarded as a maximum effect of such causes. This would

correspond at the equator to a displacement of about 0.001 Ångström, decreasing to about two-thirds of this amount at the limb. A combination of the above error in centering with an error (very extreme) of 1° in orientation, would involve a displacement of about 0.0013 Ångström at the limb. In either case, it is evident that the differential measures of the present investigation would not be appreciably affected by this cause.

4. Circularly polarized light, falling on the silvered surfaces of the two coelostat mirrors, becomes elliptically polarized, and incident light elliptically polarized has its ellipticity modified. While it can be shown that the polarization produced in unpolarized incident light by metallic reflection cannot cause displacements of the kind observed, it is quite possible that the change in the ellipticity of polarized light might be great enough to affect the displacements. For the rapid and convenient study of the change in ellipticity corresponding to any given declination and hour angle of the sun, a special polarimeter has been constructed in the Observatory instrument shop. This is essentially a small model of the tower telescope, provided with polarizing and analyzing attachments (Plate IV, *b*). At the upper end of the polar axis of the model coelostat an adjustable bronze arc, carrying a small incandescent lamp, is attached. By varying the length of the arc the incandescent lamp can be made to coincide in direction with the sun at any declination. Thus rotation of the polar axis will cause the incandescent lamp to move through a path corresponding to the path of the sun in the heavens, for the date in question. The light of the incandescent lamp, rendered parallel by a collimating lens, falls upon a Nicol prism and subsequently upon a quarter-wave plate, set so that its principle section makes an angle of 45° with the short axis of the Nicol. By setting the principal section to the left or right, right-handed or left-handed circularly polarized light can be produced at will. After falling upon the coelostat mirror the light is reflected to the second mirror and then to a small telescope, over the object-glass of which is mounted a quarter-wave plate and a Nicol prism. After the instrument has been set for the proper declination and hour angle, it is then only necessary to turn the quarter-wave plate and the Nicol prism on the observ-

ing telescope until the light of the incandescent lamp is completely extinguished. The position of the principal section of the quarter-wave plate and the major axis of the ellipse, as given by a divided circle, furnish the data required. It should be added that since the quarter-wave plate is corrected for a particular wave-length, it is necessary to place a piece of colored glass over the incandescent lamp in order to secure nearly complete extinction.

It is evident that the displacements of the polarized components of a solar line will not be affected unless the ellipticity of the light is altered sufficiently to affect the degree of its extinction, and consequently the center of gravity of the resultant line. As a convenient check on the performance of the compound quarter-wave plate and Nicol under the exact conditions of observation, the polarimeter is so constructed that these can be mounted for use with it (see Plate IV, *b*). In case the extinction is incomplete, a half-wave plate, inserted between the compound quarter-wave plate and Nicol, is rotated until the intensity of the light transmitted by alternate strips of the quarter-wave plate is reduced to a minimum.

For the preliminary investigation of the sun's magnetic field described in this paper, the compound quarter-wave plate and Nicol, used without an intervening half-wave plate, have been found to give sufficiently complete extinction for all declinations of the sun, provided the hour angle is not too great. In the definitive investigation soon to be undertaken, small corrections to the observed displacements may become necessary, but these will not affect the general conclusions.

An examination of the matter from a theoretical standpoint leads to a similar conclusion. The expression for the resultant displacement, including the effect of the change in ellipticity, indicates that the mirror polarization is such as to make the observed displacements slightly smaller than they would otherwise have been. For any given conditions of incidence, the change in ellipticity flattens but does not modify the general character of the curve or shift the maximum and minimum in latitude.[1]

5. Under certain conditions, the polarization produced by the slit and grating of a spectrograph may profoundly influence the

[1] Seares, *Contributions from the Mount Wilson Solar Observatory*, No. 72; *Astrophysical Journal*, **38**, 99, 1913.

relative intensities of spectrum lines due to light initially polarized.[1] The spectrograph polarization was examined by Mr. Ellerman by photographing the solar spectrum successively upon the same plate with a constant exposure, a Nicol in front of the slit being rotated 22°.5 between the successive exposures. The λ 5930 region of the third order shows no appreciable variation of intensity with the rotation of the Nicol, from which it appears that for this order and region the polarization must be very small. The corresponding region for the second order shows, however, a strong effect, the relative reduction in intensity for two positions of the Nicol differing by 90° amounting apparently to about 40 per cent. It happens, however, that the position of the Nicol for the most favorable transmission corresponds approximately to that of the long Nicol used in the observations for the general field, so that even here there can scarcely have been any changes in intensity. But admitting even the presence of such a change, it is difficult to see how the relative displacements could have been seriously modified. For under the conditions of observation, the light entering the slit from all the components of the solar line is plane-polarized in the same plane, owing to the presence of the Nicol in front of the slit which transmits only those vibrations parallel to the slit. The spectrograph polarization cannot, therefore, change the relative intensities of the components in any spectrum strip, and, hence, apparently cannot displace the maximum of the resultant line. In the case of a very unsymmetrical resultant line, however, which does not occur with a normal triplet for Zeeman separations of the magnitude of those observed, it is conceivable that a small apparent relative shift in two adjacent spectrum strips might be introduced. But the displacements of such a line would also be affected by the conditions of exposure and development, and would be likely to present anomalies revealing its presence.

6. Two compound quarter-wave plates have been used for the work on the lines λ 5812, λ 5828, and λ 5930: one by Werlein, corrected for λ 6300, and one made in our laboratory by Mr. Babcock, corrected for λ 5650. For the latter, as tested by Mr. Babcock,

[1] Zeeman, *Proceedings of the Royal Acad. of Sciences of Amsterdam*, November 28, 1907; November 8, 1912.

the intensity of the component totally quenched at λ 5650 is 1.6 per cent at λ 6500, 0.6 per cent at λ 5930, and 2.3 per cent at λ 5100. For the Werlein plate, assuming the same rate of divergence, the intensity of the weak component at λ 5930 would be 0.8 per cent of the intensity of the strong component.

DISCUSSION OF THE OBSERVATIONS

An examination of the tables and curves representing the first, third, and fourth series of observations, all of which were made in the third-order spectrum, shows at once a marked grouping of positive displacements in the northern and of negative displacements in the southern hemisphere (quarter-wave plate in the normal or + position), with values decreasing, on the average, from middle latitudes toward the equator. The fourth series, which includes the higher latitudes, also shows a decrease toward the poles. There are various discrepancies of sign, some of which may represent mistakes of record. For example, Plates 26 and 27 may have been made in the northern hemisphere, though recorded as in the southern, since both Miss Lasby and Mr. Van Maanen obtain values which give positive mean displacements (see Tables IV, V, and VI).

As a further illustration of possible mistakes in record, Plates 279*b* and 280*a*, both made in the southern hemisphere with the quarter-wave plate in the + position, show positive values of the displacement for both lines at all of the latitudes measured (see Table IX, latitudes S. 24, 28, 34, 37, 47, 51, 59, 65, 72, 78).

But quite irrespective of these discordances, and notwithstanding the fact that there are systematic differences in the results for different lines and series, a variation of the displacement with the latitude is clearly shown in all cases excepting for λ 5930 in the second series. If in Miss Lasby's measures for the first series, including 103 values, we disregard the 10 discordant displacements in parentheses in Tables IV and VI, the results are remarkably consistent. Of the 10 discordant values, 6 are from the two plates, Nos. 26 and 27, referred to above. The remaining 4 are from Plates 14 and 18. It may be noted that Mr. Van Maanen's measures of the same region on Plate 14 are also positive, though

for latitudes S. 46° and S. 65° on the same plate he obtains the negative sign (Table VI).

We have already considered briefly the question of systematic errors (p. 63), which may depend upon both personal equation and on the very different types of measuring machines employed by the two observers. As soon as the machine for the study of systematic errors is completed, this point will be fully investigated. The present problem is to detect the sun's magnetic field and determine its polarity; the measurement of its intensity over a wide range of level and the accurate location of the poles may follow.

We have still to consider, however, the difference between the results of the second and third orders, and the peculiar behavior of λ 5930, which shows little, if any, change of displacement with latitude on the second-order plates. The first question that presents itself is whether any fundamental distinction exists between the spectra of the second and third orders, which might account for the observed effects. The influence of polarization in the spectrograph, which usually varies from one order to another, has already been discussed (p. 72), and it seems unlikely that this can have played any part. Differences in contrast and photographic structure of the lines may afford a possible explanation. I accordingly asked Dr. Anderson to make a microscopic examination of the three lines in question on plates of the second and third orders. It is a well-known fact that in a region where the density of the images changes rapidly, the silver grains are not uniformly distributed. They tend to group themselves in irregular lines or patches with lanes between them in which few, if any, dark silver grains are present. This phenomenon was investigated by Perrine in the case of star images and reproductions of enlarged photographs accompany his papers.[1] The width of the region of irregular distribution for λ 5930 was found to vary between 0.06 mm and 0.09 mm on second-order plates, while on third-order plates its width is from 0.08 mm to 0.012 mm. One striking difference, however, was noted, which may perhaps be due to the smaller contrast of the third-order plates: the distribution of the silver grains over the edges of the line is very much more uniform in the

[1] *Loc. cit*

third than in the second-order plates. In fact, on some of the plates the distribution is so nearly uniform that there was some difficulty in locating "the region of irregular distribution." From Perrine's results it would appear that such a difference would have a very important effect upon the measures, and might possibly account for the difference found between the two orders. To test this point, plates with less contrast will be made in the second order.

The differences between λ 5930 and $\lambda\lambda$ 5812 and 5828 indicated by the first and second series is perhaps rendered questionable by the fact that Mr. Van Maanen's numerous measures on $\lambda\lambda$ 5812 and 5828 in the fourth series give a curve whose maximum is practically identical with that for λ 5930 in both the first and third series. But, a real difference in displacement would in nowise be surprising, for the lines are of different intensities ($\lambda\lambda$ 5812 and 5828, intensity 0; λ 5930, intensity 2) and may represent different solar levels. It is also quite possible that their Zeeman separations for the same field strength are different.

In the early part of this investigation, when there was difficulty on the part of some observers in checking Miss Lasby's results, it occurred to me that a simple means might be employed to determine the nature of the displacements. If they are really magnetic, rotation of the Nicol prism through an angle of 90° should change a positive displacement into a negative one. As already remarked, the effect of turning the Nicol is accomplished in the present apparatus by rotating a half-wave plate, mounted between the Nicol and the compound quarter-wave plate. When the principal section of the half-wave plate is set so as to coincide with the long axis of the Nicol, the displacements of the solar lines should be the same as when no half-wave plate is used. Rotation of the half-wave plate through an angle of 22°5 should annul the displacements and cause the sections of the solar lines, in all of the successive strips, to lie in their normal position. Further rotation of the half-wave plate to a point where its principal section makes an angle of 45° with the axis of the Nicol, should restore the displacements, but their signs should be opposite to those observed when the half-wave plate stood at 0°. The atmospheric lines should not be affected, as they are not produced in a magnetic field.

The photographs with which the half-wave plate was used are indicated in the last column of Table III. When necessary, the signs of the corresponding displacements have been reversed to reduce them to the normal or zero position before inserting them in the tables containing the detailed results. Those corresponding to the 22°.5 position were entered without change, and attention called to the fact that they should be zero (see footnotes, Tables V and VI).

The values of the displacements found by Miss Lasby, with their original signs, are collected in Table X, the unit, as usual, being 0.001 mm. The latitudes for the plates compared are the same. As a check upon any abnormality in the half-wave plate, a different part of the plate was used for Plates 35–39.

TABLE X

REVERSAL OF SIGN WITH ROTATION OF HALF-WAVE PLATE

Plate Nos	26, 27		29, 30		32, 33, 34			35, 36		37, 38, 39		
λ/2 Plate	0°	−45°	0°	−45°	0°	−22°.5	−45°	±180°	−135°	+135°	+157°.5	±180°
Atm. line	0	0	...	...	...	0	0	0	0	0	n.m.	0
λ 5812...	+ 8	−4	+8	−6	...	...	...	...	...	...	....	...
λ 5828...	+12	−4	+8	−3	...	...	...	...	...	...	....	...
λ 5930...	+ 8	−1	...	−1	+9	+3	−4	+7	−7	−5	n.m.	+7
Means...	+9	−3	+8	−3	+9	+3	−4	+7	−7	−5	....	+7

These results clearly show that the magnitude of the displacements depends upon the position angle of the half-wave plate, and that their direction can be reversed by turning the plate through an angle of 45°. We may therefore conclude that the light from the red and violet sides of the solar line in question is circularly or elliptically polarized in opposite directions.

Another method of accomplishing the same result is the inversion of the compound quarter-wave plate, now regularly employed in all of our observations. Thus the relative signs of the displacements on the *a* and *b* plates of the second series should afford another check on their magnetic nature.

As previously explained, the columns in Tables VII–IX headed + and − give the observed displacements and the signs obtained

when the quarter-wave plate was in the normal and inverted positions. If the displacements were large, and the source were actually in a magnetic field, the sign of the displacement corresponding to the + and − positions of the quarter-wave plate would always be reversed. In a comparatively weak magnetic field, however, giving very small displacements, the errors of measurement would necessarily tend to mask the effect, and in many cases the expected reversal of sign would not be observed. Table XI indicates the percentage of cases in which a reversal occurs. Those instances in which one of the displacements is zero have been disregarded in calculating the percentages, the number of such cases being indicated in the column headed "Zero Values"; but discordant observations have not been rejected in the comparison.

TABLE XI

REVERSAL OF SIGN WITH INVERSION OF QUARTER-WAVE PLATE

Series	λ	No. Reversed	No. Not Reversed	Rev. + Not Rev.	Zero Values	Percentage Reversed
II.....	5812	13	5	18	2	72
II.....	5828	13	7	20	1	65
II.....	5930	27	9	36	5	75
III.....	5930	32	8	40	0	80
IV.....	5812	21	6	27	0	78
IV.....	5828	24	3	27	0	89
Total...		130	38	168	8	77

From an examination of the table, we find that 130 pairs of plates out of 168, or 77 per cent, show reversal of sign. In view of the relatively large errors of observation, this result strongly supports the hypothesis that the displacements are produced by a magnetic field.

The effect of inverting the quarter-wave plate is graphically shown in Fig. 4, where the measures corresponding to the + and − positions of the plate are plotted separately. Two nearly symmetrical curves result, with opposite slopes, just as theory would indicate in the case of a magnetic field. A similar application of the same method is illustrated by the two curves shown in Fig. 5.

It is important to note that the percentage of reversals for

λ 5930 in the second series is the same as that for $\lambda\lambda$ 5812 and 5828, in spite of the fact that the former line shows, in this series, no

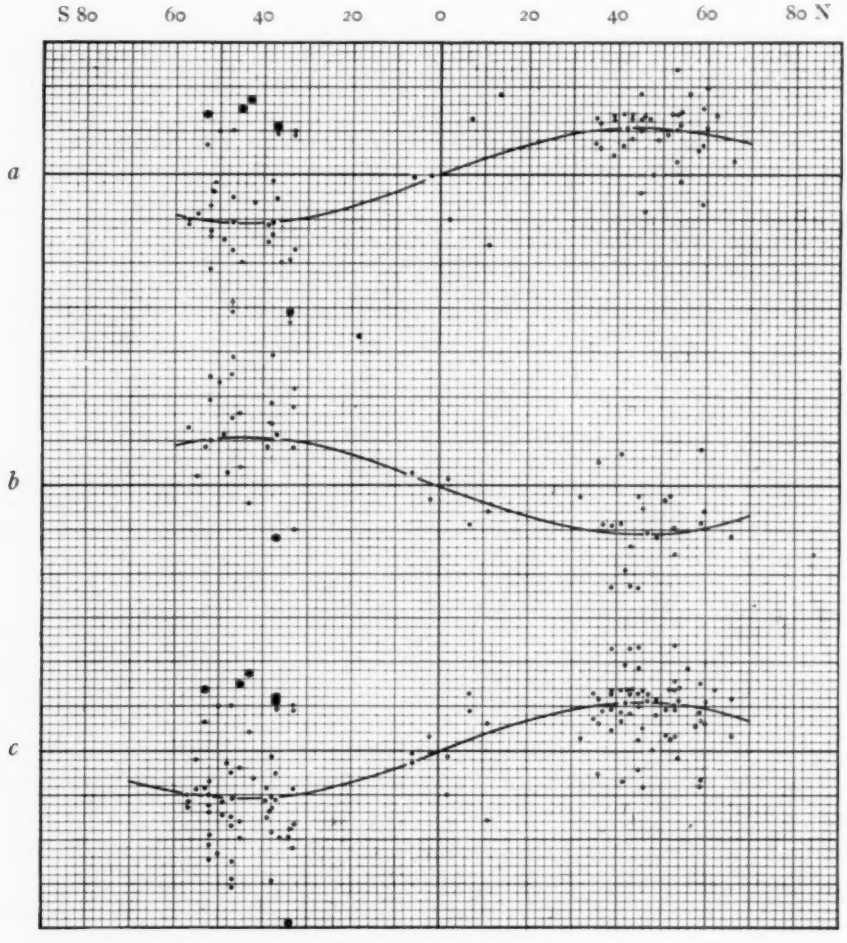


FIG. 4.—Displacements for the third series, λ 5930, measured by Mr. Van Maanen

a, Quarter-wave plate in normal position; *b*, Quarter-wave plate inverted; *c*, Results for *a* and *b* combined, the signs of the displacements for the latter having been changed.

Vertical scale: 1 division = 0.001 mm.

variation of the displacement with the latitude, while for $\lambda\lambda$ 5812 and 5828 it seems clear. This points strongly to the conclusion

that the apparently discordant results for λ 5930 are really magnetic. I am unable to offer any explanation of this anomaly, though attention should be called to the fact that two spots appeared on the sun while the second series was in progress.

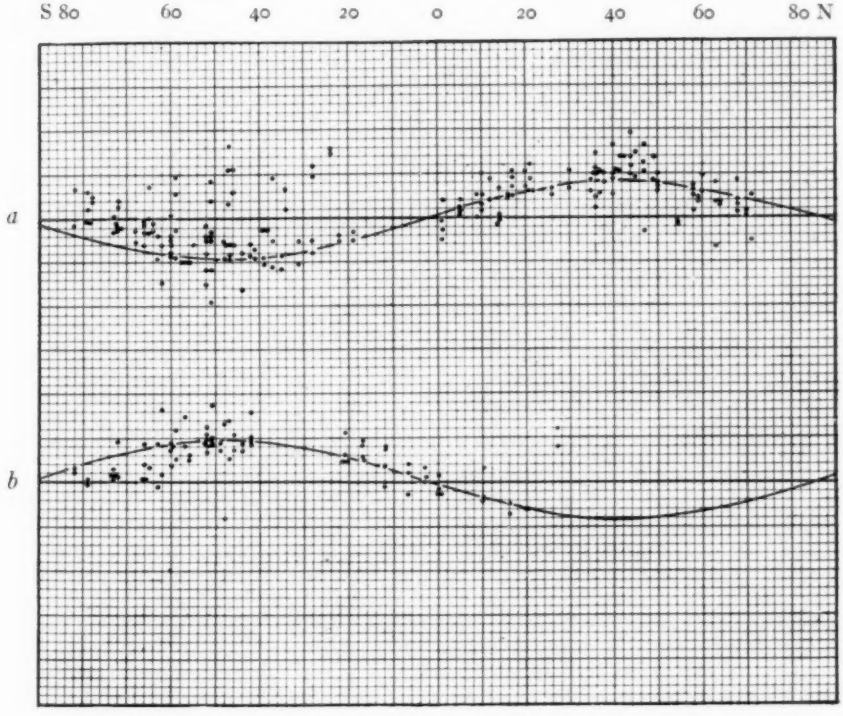


FIG. 5.—Displacements for the fourth series, λλ 5812 and 5828, measured by Mr. Van Maanen.

a, Quarter-wave plate in normal position; *b*, Quarter-wave plate in inverted position.

Vertical scale: 1 division = 0.001 mm.

The evidence would thus appear to be conclusive that, within the precision of measurement, the sign of the displacement is reversed by inverting the quarter-wave plate. As atmospheric lines fail to show this effect, it cannot be considered of nstrumental origin. Furthermore, many solar lines do not share in the displacements. We are thus led to the conclusion that the cause

of the displacements is a magnetic field of sufficient strength to produce the observed displacements of λ 5912, λ 5828, and λ 5930, but too weak at certain other levels in the solar atmosphere to affect the lines which represent them.

The analogy afforded by sun-spots, where I have found a rapid decrease in the strength of the field in passing from low to high levels,[1] suggests that we are concerned with a low-level phenomenon. This view is supported, especially in the case of λ 5812 and λ 5828 by the results of Mr. St. John's recent investigations, which show that, in general, the fainter lines of the solar spectrum represent the lower levels. Another check is afforded by some measures made for me by Mr. Adams, who finds the following displacements at the sun's limb for the three lines in question:

λ 5812	0.012 Ångström
λ 5828	0.011
λ 5930	0.011

In his investigation on the displacements of lines at the sun's limb, Mr. Adams concluded that the displacements are largest for the elements which lie at the lowest levels in the sun's atmosphere.[2] Since the displacements given above are large for this region of the spectrum, we may conclude that the lines in question are probably produced at low levels. It should be noted, however, that other solar lines, showing equally large displacements at the limb, apparently give no evidence of the Zeeman effect. Mr. St. John will soon test this point further by a direct determination of the pressure displacements.

HYPOTHESIS OF LOCAL WHIRLS

The evidence presented above seems sufficient to prove that the observed displacements are caused by magnetic fields in the sun. We may next consider whether these fields are due to local phenomena or represent the magnetic effect of a rotating sphere. We know that sun-spots always show the Zeeman effect, and that

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 30, p. 15; *Astrophysical Journal*, **28**, 329, 1908.

[2] *Contributions from the Mount Wilson Solar Observatory*, No. 43, p. 25; *Astrophysical Journal*, **31**, 30, 1910.

the widening of the lines frequently extends beyond the boundary of the penumbra. Magnetic fields may also be caused by invisible spots or by whirls in which no umbrae or penumbrae have appeared, if we may judge from the structure frequently presented by the H_α flocculi. Finally, there is some evidence to support the view that the pores are small vortices, which develop into spots under favorable conditions. Is it possible that the observed displacements are due to any of these causes?

I believe that we may answer this question without hesitation in the negative for the following reasons:

1. Our observations of sun-spots indicate that right-handed and left-handed whirls are about equally common in the northern and southern hemispheres. The great majority of spots consist of two principal members, frequently attended by satellites, the line joining the chief spots usually making a small angle with the solar equator. In general, I find these groups to be of the bipolar type, i.e., the two principal spots are of opposite polarity. When a new spot appears, it is very frequently double, and of the bipolar type. Hence there is no reason to suppose that the influence of spots, visible or invisible, incipient or disintegrating, could be of such a character as to produce Zeeman displacements which, on the average, are of opposite sign in the northern and southern hemispheres.

2. The observations have been made during a low minimum of solar activity, and in the great majority of cases no spots whatever, and few K_2 flocculi, were visible on the sun.

3. If the pores are electric vortices, like the spots, there is no reason to suppose that pores of one polarity preponderate in the northern hemisphere, and those of opposite polarity in the southern.

4. Even if there were a clear preponderance of pores of opposite sign north and south of the equator, it would be difficult to account for such a curve of displacements as the plotted observations represent.

5. Assuming, however, that such a curve could be plausibly explained as originating in the pores, it is evident from the character of the curve that we should be dealing with a general magnetic field of the sun, though not one caused by the solar rotation.

Although it is very improbable that the general curve of displacements is due to local vortices, we may expect to find irregularities in the curve when spots or other local fields lie on the slit. It is doubtful, however, whether any displacements of the wrong sign in the present investigation can be ascribed to such causes, though an exceptional number of discordant observations on June 3 and 4 may possibly be due to the proximity of a large K_2 flocculus. As the sun becomes more active, the effect of local perturbations will doubtless increase. A method of detecting them, which may prove to be useful, is to compare the area of the flocculi within a square, say, 10° on a side, centered on the slit, with the average deviation from the mean of the observed displacement.

THEORETICAL CURVE OF DISPLACEMENTS

The general expression for the displacement of a normal Zeeman triplet originating a source lying in the general magnetic field of the sun, here assumed to be a spherical magnet, is[1]

$$k\Delta = \epsilon \cos \gamma'. \tag{1}$$

The factor ϵ in the right-hand member of this equation represents the effect of the elliptical polarization produced by reflection from the silvered surfaces of the coelostat mirrors, while $\cos \gamma'$ is a function of the heliocentric latitude of the point observed, the deviation of the observer from the plane of the sun's equator, and the sun's magnetic elements. As previously stated, the effect of the mirror polarization is only to flatten the curve. The numerical values of ϵ corresponding to different values of the hour angle and declination may be seen by a reference to the table on p. 25 of Mr. Seares's paper. As the factor ϵ is nearly unity for normal observing conditions, we may disregard it for the present and consider only the quantity $\cos \gamma'$. The expression for this is

$$\begin{aligned} \cos \gamma' = &\{\tfrac{1}{3} \sin (2\phi - D) + \sin D\} \cos i + \\ &\{\tfrac{1}{3} \cos (2\phi - D) + \cos D\} \sin i \cos \lambda \end{aligned} \tag{2}$$

[1] The complete series of formulae derived by Mr. Seares, superintendent of the Computing Division, for the reduction of observations of the sun's general field, may be found in "The Displacement Curve of the Sun's General Magnetic Field," *Contributions from the Mount Wilson Solar Observatory*, No. 72; *Astrophysical Journal*, **38**, 99, 1913.

in which

ϕ = heliographic latitude of point observed,

D = heliographic latitude of sun's center,

i = inclination of magnetic axis to solar axis of rotation,

λ = longitude of north magnetic pole measured west from central meridian.

The observations show that i must be small, if not actually equal to zero, and for a preliminary comparison of the observations with the theory, we assume $i=0$. We therefore write

$$k\Delta = 3 \sin(2\phi - D) + \sin D . \tag{3}$$

The quantity k is a constant depending upon the units involved, the Zeeman separation of the line observed, and the sun's equatorial field-strength.

Since D is small, never exceeding $7°$, it follows that Δ is very nearly proportional to $\sin 2\phi$; and to the degree of approximation expressed by (3), the theoretical displacement-curve is that of $3 \sin 2\phi$ displaced laterally by the amount $\frac{1}{2} D$ and vertically by $\sin D$. These shifts are so small that they might well have been disregarded, but as they are easily included, they were taken into account in drawing the curves. This was possible, since the observations for the various series cover intervals of time so short that D may be regarded as constant for any given series.

Owing to the fact that the sun's field-strength and the Zeeman separations for the lines observed are at present unknown, it is necessary to determine the constant k from the observations themselves.

For numerically equal latitudes north and south we easily find from (3)

$$k = \frac{6 \sin 2\phi \cos D}{\Delta_n - \Delta_s} \tag{4}$$

which may be used for the determination of k. Equation (4) is most satisfactorily employed for latitudes near $45°$, and it is quite sufficient to write $\cos D = 1$.

The curves resulting from (3) and (4) corresponding to the various series of observations are shown in Figs. 2–6, together with the observations themselves. Owing to the systematic difference

between Miss Lasby and Mr. Van Maanen it was necessary to treat their results separately, and in general the displacements for each line have been considered separately for the different series. In the case of the first and third series, however, Mr. Van Maanen's results for λ 5930 have been combined for the determination of k, as an inspection of the plotted displacements showed that the values of the constant would be sensibly the same. A similar procedure was followed with $\lambda\lambda$ 5812 and 5828 in the second series.

Generally speaking, the agreement with the theory is as good as can be expected, in view of the large accidental errors of observation. There is a small irregularity in Mr. Van Maanen's results on λ 5930 for the first series (Fig. 2, *e*), but this apparently is to be attributed to the difficulties of measurement and the lack of extended experience in the use of the parallel plate micrometer, for Miss Lasby's measures of the same plates show a close agreement with the theoretical curve (Fig. 2, *d*). The results for the fourth series (Figs. 5 and 6) are of special interest, as they are more widely distributed in latitude than any of the others.

The observations of the first three series were confined to the latitudes N. 60°–S. 60°. Displacements having been found for the lower latitudes, it was desirable to make tests near the poles. For work in this region a large solar image is necessary, as the group of strips of the compound quarter-wave plate included in the measurement of the displacement for a single latitude cover at least 8 mm on the central meridian, and with a small solar image the change in latitude is too rapid to permit the four or more sets of measures to be combined satisfactorily. The 43-cm image, however, is large enough for study in very high latitudes, and accordingly a fourth series of observations was undertaken as a rigorous test of the validity of the conclusion that the displacements are due to the sun's general field. The results indicate clearly the decrease in the displacement with increasing latitudes beyond $\phi=45°$, and its approach to zero values at points near the poles. The close conformity with the theoretical curve is very satisfactory.

With regard to the theoretical expression for the displacement-curve, it may be remarked that, although developed primarily

for the case of a normal Zeeman triplet, it holds also for lines of more complicated structure including a single group of *p*-components, provided that the total intensity of the *n*-components

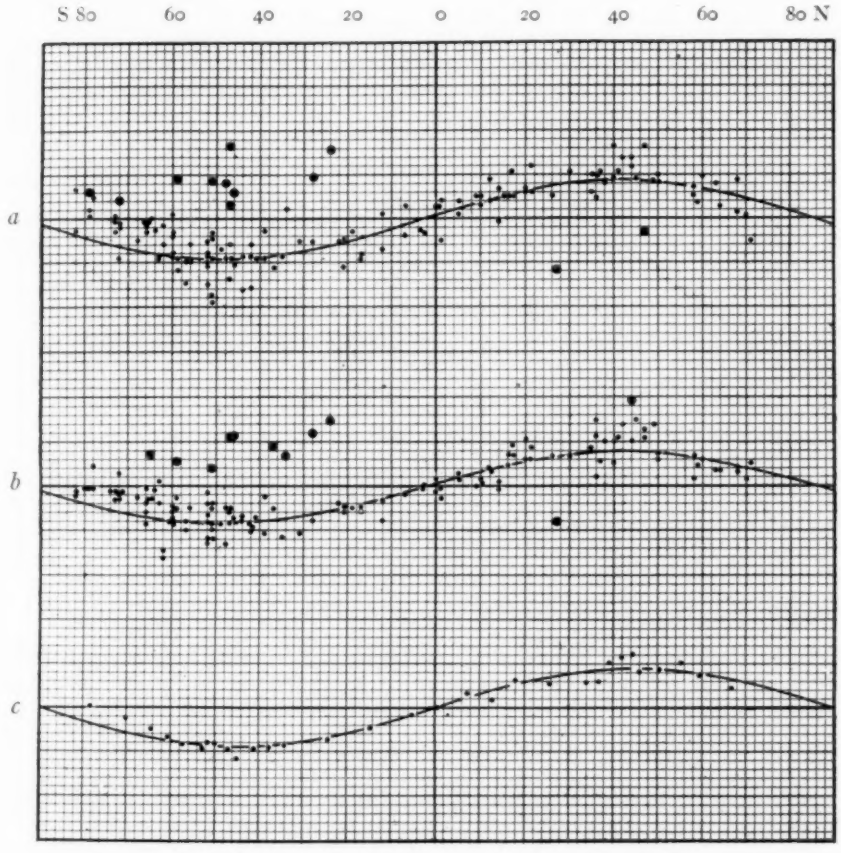


FIG. 6.—Displacements for the fourth series and mean curve

a, λ 5812; *b*, λ 5828; *c*, Mean curve of displacements including λ 5930 from first and third series and λλ 5812 and 5828 from fourth series. Measures by Mr. Van Maanen.

Vertical Scale: 1 division = 0.001 mm.

perpendicular to the field is the same as that of the *p*-components, and that the intensity distribution in the groups of *n*- and *p*-components is symmetrical with respect to the centers of the groups.

Although these conditions may not be rigorously fulfilled, laboratory results indicate that for many complex lines the approximation probably will be sufficient.

In grouping the results for the derivation of the values of k the question arose as to what should be the basis for the rejection of discordant observations—should obviously discordant plates, such as Nos. 26 and 27, be rejected as a whole, or should only those latitudes showing excessive deviations be excluded? Owing to the difficulty of establishing a satisfactory criterion for the rejection of plates, the latter alternative was adopted. The rejection in forming k was guided by a preliminary investigation of the average deviation from the mean curve defined by the observations themselves. A subsequent examination of the deviations from the theoretical curves showed, however, that the rejection was in each instance justified.

Table XII gives in a collected form the average deviations from the theoretical curves, first, on the basis of all of the material, and second, after the exclusion of certain results. Here the rejection was by Peirce's criterion, the limit being fixed by the average deviations based on all of the observed displacements. The discordant values thus found are indicated by parentheses in Tables IV–IX and by circles inclosing the points in the figures. The systematic difference between Miss Lasby and Mr. Van Maanen leads occasionally to an apparent contradiction, as, for example, in the case of latitude S. 56°, Table VI. The smaller of two positive displacements is rejected while the larger is retained. The corresponding deviations, however, are +11 and +7.8, one of which lies above the rejection limit, while the other is below it.

Table XII also gives, in the third and fourth columns, the values of the ordinates of the curves for $\phi=45°$, which are connected with k by the relation $\Delta_{45}=3/k$. These exhibit numerically the systematic differences in the various results.

For those cases in which the displacement of the curve due to the deviation of the observer from the sun's equator has been included, the values are the means of the maximum ordinates north and south, and, strictly speaking, do not correspond accurately to 45°. The zero value of the ordinate at 45° for λ 5930

in the second series is a numerical expression of the fact that this particular series shows no appreciable variation of the displacement with the latitude. The deviations in this case are referred directly to the axis, and the mean deviation is of course the average of the observed displacements without regard to sign.

The general consistency of the material is illustrated by the fact that out of 766 values only 62 are sufficiently discordant to call for rejection. These numbers do not include the measures on λ 5930 in the second series; the measures by Miss Lasby and Mr. Van Maanen in the first series have been counted as separate observations.

TABLE XII

VALUES OF DISPLACEMENT AT $\phi=45°$ AND AVERAGE DEVIATIONS

Series	λ	Δ at 45°		Miss Lasby					Mr. Van Maanen				
		L.	V. M.	All Observations		No. Rejected	Revised		All Observations		No. Rejected	Revised	
				Av. D.	No. Obs.		Av. D.	No. Obs.	Av. D.	No. Obs.		Av. D.	No. Obs.
I	5812	10.3	8.6	8.8	28	6	3.6	22	2.6	28	3	2.6	25
	5828	10.3	6.7						3.7	39	2	3.7	37
	5930	9.7	5.3	3.4	75	4	2.6	71	3.5	102	8	2.8	94
II	5812	..	4.0	..	..	..	..	..	4.1	42	3	3.5	39
	5828	..	4.0	..	..	..	..	..	3.8	42	4	2.8	38
	5930	..	0.0	..	..	..	..	..	3.4	90	..	..	..
III	5930	..	5.3	..	..	..	..	..	4.0	135	9	3.4	126
IV	5812	..	4.4	..	..	..	..	..	1.9	137	12	1.3	125
	5828	..	4.0	..	..	..	..	..	1.7	138	11	1.2	127

It is obviously useless to attempt any combination of the results into a final mean curve until the question of systematic errors shall have been investigated. It is, nevertheless, of interest to consider the mean curve defined by the measures by Mr. Van Maanen on λ 5930 in the first and third series and on $\lambda\lambda$ 5812 and 5828 in the fourth series, for all of which the systematic differences are small. The displacements (excluding those in parentheses) were arranged in order of decreasing latitude and combined into the normal points shown in Table XIII. The agreement of these

with the theoretical curve determined by the average of the ordinates at 45° for the four curves is shown in Fig. 6, *c*.

With the completion of the measures of $\lambda\lambda$ 5812 and 5828 on the plates of the fourth series, a search for other lines showing displacements was undertaken by Mr. Van Maanen. The results for the nickel line λ 5831 given in Table II suggested the desirability of systematic measures on this line. This was rendered the more important by the fact that in the meantime Mr. Babcock had succeeded in obtaining in the laboratory a value of the Zeeman

TABLE XIII

MEAN DISPLACEMENTS FROM λ 5930, FIRST AND THIRD SERIES, AND $\lambda\lambda$ 5812 AND 5828, FOURTH SERIES

Measures by Mr. Van Maanen. Unit = 0.001 mm

Lat.	Δ	No. Obs.	Lat.	Δ	No. Obs.
N 68°.8	+2.2	15	S 5°.1	−0.8	15
59.4	+3.6	15	14.3	−2.4	15
55.4	+5.1	16	24.2	−3.6	15
50.3	+4.4	15	34.1	−4.3	15
46.1	+4.1	15	37.7	−4.6	15
44.5	+6.0	15	41.0	−4.7	15
41.9	+5.7	15	45.1	−5.8	16
39.3	+5.1	15	47.1	−4.7	15
36.7	+3.0	15	49.9	−4.1	15
33.9	+2.9	15	51.7	−4.1	15
25.7	+2.7	15	52.8	−4.7	15
18.0	+3.1	16	57.3	−4.1	15
12.7	+0.8	15	60.5	−3.5	15
7.3	+1.6	15	64.5	−2.5	15
N 0.5	0.0	15	70.0	−1.1	15
			S 78.1	+0.1	15

separation for the line in question. Mr. Van Maanen accordingly turned his attention to λ 5831 and has measured all of the plates of the fourth series except Nos. 279*b* and 280*a*, which showed serious discordances in the case of $\lambda\lambda$ 5812 and 5828. Although it is not feasible to include the detailed numerical results in this paper, a graphical representation of the measured displacements, about 120 in all, is shown in Fig. 7. The maximum displacement at 45° is obviously smaller than that found for the other three lines, but the reversal of sign in passing from one hemisphere to the other is clearly indicated.

SIGN OF THE DOMINANT CHARGE AND POLARITY OF THE SUN

We have seen that within the limits of precision, the observations agree satisfactorily with the curve representing the displacements of a normal Zeeman triplet originating in a source on the surface of a magnetized sphere, and observed from a point in or near the plane of its equator. The curve is approximate, since it takes no account (1) of possible deviations from the conditions presupposed in the development of the theoretical curve; (2) of the elliptical polarization produced by the coelostat mirrors; (3) of the fact that the field is too weak to produce complete separation of the components, thus involving abnormal Zeeman effects;[1]

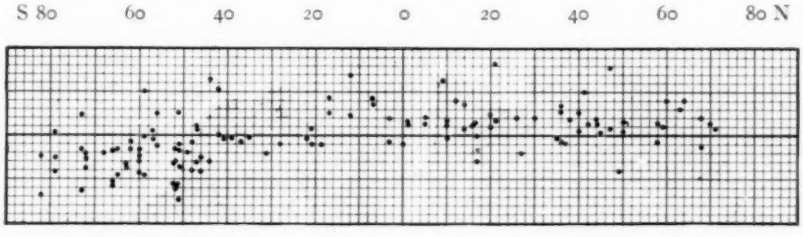


FIG. 7.—Displacements for λ 5831 from the fourth series. Measures by Mr. Van Maanen. Vertical scale: 1 division = 0.001 mm.

(4) of the possible lack of coincidence of the magnetic poles with the poles of rotation. Further, it is assumed that the spectrograph polarization is eliminated from the displacement, which apparently must be the case unless there is a wide divergence from the conditions referred to under (1). It may be shown, however, that the errors arising from these sources would be too small to be appreciable in the case of observations of the present degree of precision. Thus the results are in harmony with the conclusion that the sun is a magnetized sphere.

Disregarding, for the reasons already stated, the hypothesis of local whirls, we are led to seek the source of the sun's magnetism in its axial rotation. We may first inquire as to the sign of the dominant charge, on the assumption that the field is due to the

[1] Zeeman, "The Magnetic Separation of Absorption Lines in Connection with Sun-Spot Spectra. I," *Proc. Royal Academy of Sciences of Amsterdam*, meetings of January 29 and February 26, 1910.

rotation of a charged body or a body composed of neutral molecules which act as though they carried a charge. The observations show that in the northern hemisphere the light of the violet component of the line widened by the field is circularly polarized in the right-handed direction. In looking at the north pole of the sun, the direction of the solar rotation is left-handed. As pointed out by König and Cornu, the violet component of a magnetic doublet, observed in the direction of the lines of force (current flowing right-handedly through the coils of the magnet), is circularly polarized in the right-handed direction. Hence the sign of the dominant solar charge would be negative (opposite to that of the current through the magnet), and the polarity of the sun must correspond with that of the earth. Thus the north magnetic pole of the sun lies at or near the north pole of rotation.

STRENGTH OF THE SUN'S FIELD

In seeking to determine the strength of the sun's magnetic field, we are handicapped by the fact that the lines measured in this investigation are so weak in the arc and the spark that their Zeeman separations can be measured only with the greatest difficulty, or not at all. Repeated trials with λ 5929.898 have failed completely to show the line in the laboratory spectrum. The fainter line λ 5812.139, also identified by Rowland with iron, has likewise given great difficulty, but Mr. Babcock has finally succeeded in obtaining two plates showing measurable Zeeman separations. This fact, together with the different behavior of the two lines in sun-spot spectra renders it practically certain that the identification of λ 5929.898 with iron is erroneous. For the present, however, we may take advantage of the fact that the line appears as a doublet on some of our photographs of the sun-spot spectrum.

Table XIV contains data obtained by Mr. Babcock from a spot spectrum plate of excellent quality, No. T 190 *N*, taken on September 10, 1908, with a Fresnel rhomb and Nicol prism (in two positions of the latter). The fourth and fifth columns give the number of components and the multiple of the normal interval corresponding to each of several lines showing Zeeman separations.

The sixth column contains the measured separation from the normal position, and the last column, the field-strength of the spot in gausses calculated from the results of Mr. King's laboratory investigation of the Zeeman effect. The field-strength shows a marked variation with the intensity of the lines, but as the mean intensity of the comparison lines is approximately the same as that of λ 5929.898, we may accept provisionally $\Delta\lambda = \pm 0.038$ Ångström as corresponding to $H = 1066$ gausses. This result is, however, affected by a large uncertainty. Mr. St. John has shown that, although in general, intensity is a measure of level, there are, nevertheless, considerable differences for lines of the same intensity belonging to

TABLE XIV

ZEEMAN SEPARATIONS OF LINES IN SUN-SPOT SPECTRUM

λ (Rowland)	Element	Intensity in Sun	No. Components	$\frac{\Delta\lambda}{a\,\lambda^2}$	$\Delta\lambda$ Spot	Weight	H Spot
5918.773	*Ti*	0	3	3	±0.055Å	3	1090
5929.898	*Fe*	2	..	...	0.038	..	....
5930.406	*Fe*	6	3?	2+	0.028	1	760
5941.985	*Ti*	∞	6	2	0.043	2	1390
5966.055	*Ti, A*?	2	3	2	0.024	1	710
5985.040	*Fe*	6	3	2.5	0.038	2	920
6039.953	*V*	0	4	3	0.052	3	1050
6064.853	*Ti*	∞	3	4	0.088	3	1330
6065.709	*Fe*	7	3	1.4	±0.014	1	560
Weighted mean field-strength of spot.....							1066

different elements. As the identification of λ 5930 with iron is probably incorrect, we have no assurance that the line in question originates at the mean level of the comparison lines listed in Table XIV.

It may be shown that the polar field-strength of the sun is

$$H_p = \frac{2\Delta_{45}}{3c}$$

in which Δ_{45} is the mean relative displacement in adjacent spectrum strips for $\phi = 45°$ north and south, and c the Zeeman separation of the line observed for a field of 1 gauss. From the above data we find for λ 5930, $c = 0.000036$ Ångström. The mean of the values of Δ_{45} found by Miss Lasby and Mr. Van Maanen for λ 5930

is 0.0075 mm=0.0015 Ångström (see Table XII). Substituting these quantities into the above expression, we find for the field-strength of the sun at its pole 28 gausses.

Turning now to $\lambda\lambda$ 5812 and 5831, the laboratory results are:

λ	$\Delta\lambda$	H	$\frac{\Delta\lambda}{d\lambda^2}$	C	Weight
5812	±0.064 Å	4000 gausses	1.0	0.000016 Å	1
	0.062	6500	0.6	0.000010	3
5831	0.320 / 0.300 \	22800	0.85	0.0000136	5

Mr. Van Maanen's measures on λ 5812 (see Table XII) give Δ_{45}=0.0042 mm=0.00086 Å., and, as a first approximation for λ 5831 (Fig. 7), Δ_{45}=0.003 mm=0.0006 Å. These lead respectively to values of 48 and 29 gausses for H_p. To make these results comparable with that of 28 gausses derived from λ 5930, they should be increased by about 60 per cent, on account of the systematic difference between Miss Lasby and Mr. Van Maanen.

The value from λ 5930 is based upon the assumption that the mean of the comparison lines corresponds to the same level in the spot as λ 5930, which probably is not the case. In view of the approximate value of Δ_{45} for λ 5831, the results for this line and λ 5812 are not necessarily discordant; but all three values are uncertain because of the unknown systematic error of measurement. Both $\lambda\lambda$ 5812 and 5831 appear to be lines of complex structure, but this can scarcely have affected the values of H_p, for substantially the same results are obtained by using the displacements for $\phi=35°$. At this point Δ is practically independent of the structure of the line. Disregarding the systematic error of measurement, the results indicate that the field-strength at the sun's pole is of the order of 50 gausses.

As already remarked (p. 80), the three lines on which attention has been concentrated in this preliminary investigation are probably produced at a comparatively low level. These clearly show the effect of the sun's field, while various higher level lines, promising from the laboratory standpoint, have hitherto failed to do so (see Table I, p. 40). Thus it is probable that the intensity

of the general field falls off very rapidly in passing upward through the reversing layer. It thus presents an interesting resemblance to the field in sun-spots, though the few data now available indicate that the decrease is much more rapid in the case of the general field.

IONIZATION IN THE SUN

As serious objections have been urged against all theories of terrestrial magnetism, it is hardly to be hoped that any one of them can be applied without modification to the sun, especially in view of its high temperature, low density, and gaseous condition. In the case of sun-spots, neutral molecules cannot produce the observed fields unless an improbable degree of centrifugal separation of the positive and negative electrons is assumed. However, an important investigation by Harker seems to be directly applicable here.

Two carbon electrodes were mounted within a carbon resistance furnace 5 mm apart, and connected externally with a galvanometer. At atmospheric pressure and a temperature of 2500° C., a current of nearly 2 amperes was observed when one electrode was cooled by temporarily removing it from the hot part of the furnace. The cooler electrode, on which much carbon was deposited, was the positive one.[1] As it has been shown by Fowler and by Adams, Gale, and myself that the vapors in sun-spots are cooler than those of the surrounding atmosphere, a flow of negative electrons should therefore take place on all sides toward the umbra. These, whirled in the vortex, may account for the strong magnetic fields observed. Perhaps the exquisite structure of the penumbra may also be due in part to the effect of the field on the moving electrons and in part to electrostatic phenomena. Harker's investigation shows that strong ionization currents, increasing with the temperature, can occur at atmospheric pressure. Mr. King has recently extended this work to higher pressures, with the following results: (1) the ionization current decreases as the pressure increases; (2) the decrease is rapid up to a pressure of four atmospheres; (3) the current is appreciable at high pressures (at least up to 20 atmospheres).

[1] Harker, "Very High Temperatures," *Nature*, July 18, 1912, p. 517.

On account of the greater mobility of the negative electrons, their tendency to flow toward regions of lower temperature, and the evidence afforded by Mr. King's experiments that solar ionization is not limited to the region of the pressure in and above the reversing layer, it is evident that the electrical and magnetic phenomena of the interior of the sun must differ radically from those of the earth. It may be remarked, however, that since the negative electrons will tend, on the average, to lie farther from the center of the sun than the positive electrons, the polarity of any general field that may thus result from the solar rotation should correspond with that of the earth's field.

In the solar atmosphere, as Arrhenius, Deslandres, and others have pointed out, we have every reason to suppose that the negative electrons lie farther from the photosphere, on the average, than the positive electrons. The rotation of the atmosphere with the sun would thus tend to set up a magnetic field, of the same polarity as that of the earth. At the base of the atmosphere this field would oppose the field due to the rotation of the body of the sun. Hence, assuming a suitable distribution of the positive and negative electrons, it may be possible to account in this way for the observed decrease in the strength of the general field at increasing distances from the photosphere. It may even turn out that the Zeeman effect observed is due to the rotation of the solar atmosphere and not to the rotation of the body of the sun.

It is evident, however, that such questions as these can be discussed to much better advantage after the strength of the general field at various levels has been measured. It will also be a matter of great interest and importance to determine whether the strength of the field undergoes appreciable variation during the sun-spot period, in harmony with the changes of solar activity. The work will be carried forward by Mr. Ellerman, Mr. Van Maanen, and myself, under an enlarged program which provides for the inclusion of lines representing a wide range of level, the investigation of systematic errors and new methods of measurement, and the study of allied solar and laboratory problems.

The work described in the present paper has been done during a period when I have been able to devote comparatively little time

to research. I have been fortunate, however, in having the cordial co-operation and assistance of several members of the Observatory staff, for which I am heartily indebted. The earlier photographs of the first series were taken by myself with the assistance of Mr. Ellerman, but most of the subsequent ones of all four series are due to the persistent and most efficient work of Mr. Ellerman alone. Mr. Kohlschütter also made a number of excellent plates. In the measurements, as the results show, the skill and experience of Miss Lasby proved to be a factor of the first importance. The measurement of the plates of all four series by Mr. Van Maanen has been carried out with great skill and unflagging interest. I am also indebted to Mr. Adams for valuable check measures, and to Mr. Kohlschütter, Miss Burwell, Miss Sheldon, Miss High, and Miss Ware for much work of measurement and computation. Mr. Seares has given me invaluable aid, which I greatly appreciate, both in the supervision of measurements and reductions, and in the preparation of this paper for the press. The important reduction formulae which he has derived are published in *Contributions from Mount Wilson Observatory*, No. 72. Mr. Babcock has rendered much assistance in the preparation of polarizing apparatus, the study of weak fields and other laboratory problems, and in connection with the work of measurement. I owe to Mr. King the results of his investigations of ionization currents at high pressures. I have also had the benefit of Dr. Anderson's aid and advice in several phases of the work.

I wish to express my thanks to Professor Schuster, Professor Zeeman, and Dr. Bauer for criticisms and suggestions based on their studies of magnetic phenomena, and to M. Cotton for valuable assistance in the design and construction of the polarizing apparatus for the 75-foot spectrograph.

DIFFICULTIES AND OBJECTIONS

The evidence presented above seems to indicate pretty conclusively that the sun possesses a general magnetic field of sufficient strength to produce the Zeeman effect in certain lines. Nevertheless, the difficulties and objections in the way of accepting such a conclusion must be recognized. The most important of these are as follows:

1. The small number of lines for which a clearly defined displacement has been established.

2. Observations of the center and limb, which afford a comparatively reliable test of level, do not indicate appreciable differences of level between the three lines observed and other lines showing apparently no displacement.

3. Two series of second-order plates for λ 5930 measured by Mr. Van Maanen fail to show the displacement found by him on other series of plates.

4. Measures of the first series by other observers fail to show the displacements found by both Miss Lasby and Mr. Van Maanen.

None of these appear to be insuperable. The evidence indicates that we are dealing with a low-level phenomenon, which implies that, generally speaking, only the fainter lines can be expected to show the displacements observed in the case of $\lambda\lambda$ 5812, 5828, 5831, and 5930. Furthermore, the quantities involved are so small that for lines of average Zeeman separation extended series of observations will be required to establish clearly the displacement. Casual measures on lines apparently promising are very likely to give negative or contradictory results. Moreover, no systematic search has yet been made for additional lines. The difficulties of measurement are so great that it seemed best to establish the reality of the effect for the lines here discussed before proceeding to a detailed examination of others.

The absence of displacements for lines shown by center and limb measures to be of the same order of level as $\lambda\lambda$ 5812, 5828, and 5930 may be due either to small Zeeman separations or to a rapid falling off in the field-strength with increasing elevation.

The difficulty experienced in the case of the second-order plates has already been discussed and possible explanations have been suggested. It cannot be said, however, that these appear to be adequate. Finally, the discordant results by various observers for the first series (including Mr. Van Maanen's first set of measures for this series) are reasonably accounted for on the basis of a lack of experience with measurements of the degree of difficulty involved, or of unfamiliarity with the instrument used.

SUMMARY

Fundamental considerations relating to the properties of matter indicate that all rotating bodies may give rise to magnetic fields. Although no theory has yet been found which completely accounts for the phenomena of terrestrial magnetism, it is probable that they result from the axial rotation of the earth. The sun, a great rotating body at a temperature which precludes the existence within it of permanent magnets, offers a means of testing the theory. The form of the corona and the motion of the prominences suggest that it is a magnet, but direct proof is lacking. An attempt was accordingly made to detect the Zeeman effect due to the general magnetic field of the sun.

Preliminary observations, made in 1908 with the 60-foot tower telescope, were inconclusive. The work was renewed with the 75-foot spectrograph of the 150-foot tower telescope in January 1912 and continued through the year, with the following results:

1. The lines λ 5812.139 (*Fe*, 0), λ 5828.097 (—, 0), λ 5831.821 (*Ni*, 1), and λ 5929.898 (*Fe*, 2) show distinct displacements not shared by atmospheric lines nor by certain other solar lines.

2. The sign of the displacements is reversed, in 77 per cent of the measures, when the quarter-wave plate, mounted above a Nicol prism over the slit of the spectrograph, is inverted.

3. The sign of the displacements is reversed when a half-wave plate, mounted between the quarter-wave plate and Nicol, is turned through an angle of 45°.

4. The sign of the displacements is opposite in the northern and southern hemispheres of the sun.

5. The maximum displacements are observed about 45° north and south of the solar equator. From this point they decrease to zero at the equator and near the poles of rotation.

6. A curve representing the displacements as a function of the latitude corresponds closely with a theoretical curve, showing the displacements of a normal Zeeman triplet observed at various latitudes in the field of a magnetized sphere.

7. In view of this agreement, and the apparent impossibility of accounting for the observed displacements on other grounds,

it is probable that they represent the Zeeman effect due to the sun's general magnetic field.

8. Assuming this to be true, we find that the magnetic poles of the sun lie at or near the poles of rotation.

9. The polarity of the sun corresponds with that of the earth, i.e., the north magnetic pole lies near the north heliographical pole.

10. On the hypothesis that the magnetism of the sun is due to the axial rotation of a body acting as though it carried a residual volume charge, the sign of the charge comes out negative.

11. The preliminary results indicate that the general magnetic field decreases rapidly in intensity at levels in the solar atmosphere higher than those represented by the lines named in paragraph 1.

12. A first approximate value for the vertical intensity of the sun's general field at the poles is 50 gausses.

MOUNT WILSON SOLAR OBSERVATORY
March 28, 1913

THE DISPLACEMENT-CURVE OF THE SUN'S GENERAL MAGNETIC FIELD[1]

By FREDERICK H. SEARES

In *Contributions from the Mount Wilson Solar Observatory*, No. 71, Mr. Hale has described the results of an investigation undertaken for the purpose of determining whether the sun possesses a general magnetic field similar to that surrounding the earth. The observational evidence seems clear. The spectrum lines observed with the 75-foot spectrograph and the polarizing apparatus of the 150-foot tower telescope show displacements that apparently cannot be attributed to any other cause. The displacement varies with the latitude, and the algebraic sign reverses in passing from one hemisphere to the other; but in order that there might be a more rigorous control of the results and their interpretation, it seemed advisable to compare the observed displacements with the theoretical displacement-curve derived on the assumption that the sun is a magnetized sphere. It was further desirable to have available formulae for determining the position of the magnetic axis relative to the axis of rotation. To satisfy these requirements the equation connecting the displacement of a spectrum line with the solar magnetic elements and the co-ordinates of the observer and the observed point was established. The following pages contain the development of this equation and an indication of its application to the determination of the position of the sun's magnetic axis. Owing to our imperfect knowledge of the electrodynamical theory underlying the structure of more complicated lines, the discussion will be restricted mainly to the case of the simple Zeeman triplet, although indications of possible extensions of the theory will be given.

Mr. Hale has fully described the instrumental equipment; but as what follows depends intimately upon the construction and operation of the polarizing apparatus, the essential features of this part of the instrument will be explained. It includes a Nicol prism

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 72.

and a compound quarter-wave plate, placed immediately in front of the slit of the spectrograph. The quarter-wave plate consists of a series of mica strips mounted perpendicularly to the slit, with their edges in contact, between two glass plates. The optical axes of the strips are inclined at an angle of 45° to the slit, with the "fast" directions of adjacent strips perpendicular to each other. The Nicol is adjusted so that plane polarized light vibrating parallel to the slit is freely transmitted, while that vibrating perpendicularly to the slit is obstructed.

If, therefore, a beam of circularly polarized light be directed toward the slit of the spectrograph, it will be transformed by the mica strips of the quarter-wave plate into a series of plane polarized beams whose vibration planes are alternately parallel to and perpendicular to the slit. The Nicol transmits the former but not the latter. The slit will therefore be illuminated throughout 2 mm intervals with dark stretches of 2 mm intervening. If a beam circularly polarized in a direction opposite to that just considered be substituted, the illuminated portions will become dark, while the dark parts will receive light.

If now a luminous source situated in a magnetic field be examined in the direction of the lines of force, the slit will be illuminated throughout its length, owing to the presence of the opposite circular vibrations emitted by the outer components of the Zeeman triplets. The spectrum will be divided longitudinally into two groups of strips, alternately distributed, one of which will show the right, and the other the left components of the triplets. Under the conditions assumed, the *p*-components do not appear; any given strip contains radiation from but one of the *n*-components, and the displacement from the normal position of the line is proportional to the field-strength. It is the distribution of the radiation and the amount of the resultant displacement for the modified conditions occurring in solar observations that must be determined. The most important modification in such observations lies in the fact that the line of sight does not in general coincide in direction with the lines of force. The circular vibrations therefore enter the analyzer as elliptically polarized light, and, in addition, a portion of the linear vibrations from the midde

component also reaches the instrument. The action of the quarter-wave plate and the Nicol upon this mixture of elliptical and plane polarized light has now to be investigated.

Let γ represent the angle included between the line of sight and the lines of force cutting the sun's surface at the point observed. Viewed at this angle the opposite circular vibrations of the triplet may be represented by

$$\left.\begin{aligned} x &= a \sin \omega_1 t \\ y &= b \cos \omega_1 t \end{aligned}\right\} \tag{1}$$

$$\left.\begin{aligned} x &= a \sin \omega_2 t \\ y &= -b \cos \omega_2 t \end{aligned}\right\} \tag{2}$$

in which $b = a \cos \gamma$. The form of (1) and (2) assumes that the axes of x and y are perpendicular to the line of sight, and oriented so that OY lies in the plane defined by the line of sight and the lines of force. The elliptical vibration (1) is equivalent to the two opposite circular vibrations

$$\left.\begin{aligned} x_1 &= \tfrac{1}{2}(a+b) \sin \omega_1 t \\ y_1 &= \tfrac{1}{2}(a+b) \cos \omega_1 t \end{aligned}\right\} \tag{3}$$

$$\left.\begin{aligned} x_2 &= \tfrac{1}{2}(a-b) \sin \omega_1 t \\ y_2 &= -\tfrac{1}{2}(a-b) \cos \omega_1 t \end{aligned}\right\} \tag{4}$$

Owing to the fact that the slit is always placed parallel to the central solar meridian, the axes of x and y will not coincide with the directions of the optical axes of the quarter-wave plate. Let β be the angle through which the axes of x and y must be rotated in the positive (counter-clockwise) direction in order to bring them into coincidence with the optical axes of any given section of the quarter-wave plate. The vibrations (3) and (4) when referred to the latter axes become

$$\left.\begin{aligned} \xi_1 &= \tfrac{1}{2}(a+b) \sin (\omega_1 t+\beta) \\ \eta_1 &= \tfrac{1}{2}(a+b) \cos (\omega_1 t+\beta) \end{aligned}\right\} \tag{5}$$

$$\left.\begin{aligned} \xi_2 &= \tfrac{1}{2}(a-b) \sin (\omega_1 t-\beta) \\ \eta_2 &= -\tfrac{1}{2}(a-b) \cos (\omega_1 t-\beta) \end{aligned}\right\} \tag{6}$$

Equations (5) and (6) may be written at once by noting that the only effect of the rotation is to increase the phase of the negative (clockwise) circular vibration (3), and decrease that of the positive vibration (4), by the angle β.

The circular vibrations (5) and (6) are transformed by the quarter-wave plate into linear vibrations whose planes, with a proper orientation of the "fast" and "slow" directions of the plate, are respectively parallel to and perpendicular to the slit. If the direction of the slit be denoted by OY', the resulting linear vibrations will be

$$\left.\begin{aligned} x' &= \frac{1}{\sqrt{2}}(a-b)\cos(\omega_1 t-\beta-\delta_1) \\ y' &= \frac{1}{\sqrt{2}}(a+b)\cos(\omega_1 t+\beta-\delta_1) \end{aligned}\right\} \tag{7}$$

The second elliptic vibration (2), gives a similar result, which is immediately derived from (7) by changing the sign of b and modifying the phase to correspond with the frequency ω_2. Thus,

$$\left.\begin{aligned} x' &= \frac{1}{\sqrt{2}}(a+b)\cos(\omega_2 t-\beta-\delta_2) \\ y' &= \frac{1}{\sqrt{2}}(a-b)\cos(\omega_2 t+\beta-\delta_2) \end{aligned}\right\} \tag{8}$$

Consider now the linear vibrations emitted perpendicularly to the lines of force by the central component of the triplet. Since a is the amplitude of the original circular vibrations, that of the linear vibrations must be represented by $\sqrt{2}\,a$. Viewed at the angle γ the latter are therefore of the form

$$x=0, \quad y=\sqrt{2}\,a\sin\gamma\cos\omega t.$$

The components parallel to the optical axes of the strip previously considered are therefore

$$\begin{aligned} \xi &= \sqrt{2}\,a\sin\gamma\sin\beta\cos\omega t \\ \eta &= \sqrt{2}\,a\sin\gamma\cos\beta\cos\omega t \end{aligned}$$

Traversing the quarter-wave plate, these are transformed into the elliptic vibration

$$\begin{aligned} \xi &= -\sqrt{2}\,a\sin\gamma\sin\beta\sin(\omega t-\delta) \\ \eta &= \sqrt{2}\,a\sin\gamma\cos\beta\cos(\omega t-\delta) \end{aligned}$$

whose components perpendicular to and parallel to the slit are, respectively,

$$\left.\begin{aligned} x' &= -a\sin\gamma\cos(\omega t-\beta-\delta) \\ y' &= a\sin\gamma\cos(\omega t+\beta-\delta) \end{aligned}\right\} \tag{9}$$

This result can also be derived directly from (7) by writing $a=0$, $b=\sqrt{2}\,a\sin\gamma$, and modifying the phase.

Equations (7), (8), and (9) represent the various types of vibration in the beam emerging from the mica strip of the quarter-wave plate. The results for an adjacent strip will be similar, but with an interchange in the x' and y' components. Each type corresponds to one of the three components of the triplet. As all vibrations parallel to the slit are transmitted by the Nicol, it appears that each type will be represented in the spectrum. Each spectrum strip will contain all three components, but with an alternation in the intensity distribution. For one set of alternate strips the intensities from violet to red will be

$$\tfrac{1}{2}(a+b)^2, \quad a^2 \sin^2\gamma, \quad \tfrac{1}{2}(a-b)^2;$$

for the other,

$$\tfrac{1}{2}(a-b)^2, \quad a^2 \sin^2\gamma, \quad \tfrac{1}{2}(a+b)^2.$$

Substituting the value $b=a\cos\gamma$, and assuming that the total intensity of the three components is unity, these expressions become

$$V=\tfrac{1}{4}(1-\cos\gamma)^2, \quad M=\tfrac{1}{2}\sin^2\gamma, \quad R=\tfrac{1}{4}(1+\cos\gamma)^2. \qquad (10)$$

The separation is proportional to the field-strength, but in the case of weak magnetic fields the lines will not be resolved. Owing to the different intensities of the components, however, the resultant blend will not coincide with the normal position of the triplet; and as the displacement in adjacent strips is in opposite directions, it will still be possible to determine the amount of the separation. It is the relative displacement of the blended line for adjacent spectrum strips that Mr. Hale has observed.

The derivation of an expression for the resultant displacement requires a consideration of the way in which the three components unite to form the resultant. The intensity distribution in the three lines may be represented by

$$y_1=k_1 f(x+\sigma), \quad y_2=k_2 f(x), \quad y_3=k_3 f(x-\sigma), \qquad (11)$$

in which σ is the separation of the outer components from the middle line. The constants k are independent of x, and we assume the functions f to be symmetrical with respect to the axis of y. The total intensities are therefore the integrals between $-\infty$ and $+\infty$ of the expressions (11). Since the definite integrals of $f(x+\sigma)$, $f(x)$, and $f(x-\sigma)$ are equal, it follows that the values of k are respectively

proportional to V, M, and R. Omitting the factor of proportionality, we may write

$$y_1 = Vf(x+\sigma), \quad y_2 = Mf(x), \quad y_3 = Rf(x-\sigma). \tag{12}$$

The equation defining the intensity distribution of the blended line is therefore

$$Y = Vf(x+\sigma) + Mf(x) + Rf(x-\sigma). \tag{13}$$

The question now arises as to what point of this line is to be regarded as determining its position. For a sharply defined maximum, there would undoubtedly be a tendency to make settings upon the maximum itself in any attempt to determine the position of the line. For a broad, diffuse line with a flat maximum, probably the center of gravity would be the determining point. The lines observed by Mr. Hale are of this latter character, but it will be shown that the result is essentially the same, whichever point be taken as measuring the displacement, as long as the separation σ is small as compared with the width of the line. This requirement, however, is satisfied in the case of observations of the sun's general field. For the lines used thus far, the ratio of separation to width is of the order of 0.01, and it will therefore be possible to simplify the discussion by disregarding the second and higher powers of σ.

Developing the first and third terms of (13) according to powers of σ and noting that by (10),

$$V+M+R=1, \quad R-V=\cos\gamma, \tag{14}$$

we find

$$Y = f(x) - \sigma\cos\gamma\, f'(x). \tag{15}$$

Equation (15) represents, however, the first two terms of a Taylor's series development, the remaining terms of which contain squares and higher powers of σ. We may therefore write

$$Y = f(x-\sigma\cos\gamma). \tag{16}$$

To the degree of precision considered, the resultant line has therefore the same form as its components, and is symmetrical with respect to

$$x = \sigma\cos\gamma \tag{17}$$

as an axis. The co-ordinates of its maximum and center of gravity coincide and are determined by (17).

Since the relative displacement in adjacent strips of the spectrum is twice the displacement from the normal position, we have

$$\Delta = 2\sigma \cos \gamma . \tag{18}$$

It will be noted that the result is independent of the particular form of the intensity distribution. We have assumed merely that $f(x)$ is symmetrical, that it is developable for small values of x, and that σ is small as compared with the width of the lines. It appears at once from a consideration of the geometrical conditions of the problem that all of these are admissible. It will be observed further that the result is independent of the orientation of the slit, for it does not contain the angle β which defines this orientation.

We have now to evaluate the two factors in (18). The first depends upon the field-strength and the dispersion of the spectrograph. Assuming the distribution of the magnetic force over the sun's surface to be similar to that on the earth, its intensity at any magnetic latitude ϕ' will be[1]

$$H = H_e H' \tag{19}$$

in which H_e is the equatorial field-strength, and

$$H' = 1 \sqrt{1+3 \sin^2 \phi'} . \tag{20}$$

We therefore have

$$\Delta = 2cH_e H' \cos \gamma \tag{21}$$

in which c is a constant determined by the units employed.

To determine the angle γ consider Fig. 1, and let it be recalled that the observations are always made with the slit in coincidence with the central meridian of the solar image.

The figure represents a projection of the various points involved on the surface of the celestial sphere, the center being at C. We have

P = sun's north pole.
P' = adjacent magnetic pole.
O = observed point, always on the central meridian PE.
E = intersection of central meridian with sun's equator.
E' = intersection of magnetic meridian through O with magnetic equator.

[1] Foster and Porter, "Electricity and Magnetism," 314, London, 1903.

S = intersection of line of sight with sphere.
F = intersection with sphere of tangent to line of force at observed point.
ϕ = EO = heliographic latitude of O.
ϕ' = $E'O$ = magnetic latitude of O.
D = ES = heliographic latitude of sun's center.
Q = angle at O between central meridian and magnetic meridian.
δ' = angle between sun's surface and line of force at observed point; $90-\delta' = FO$.

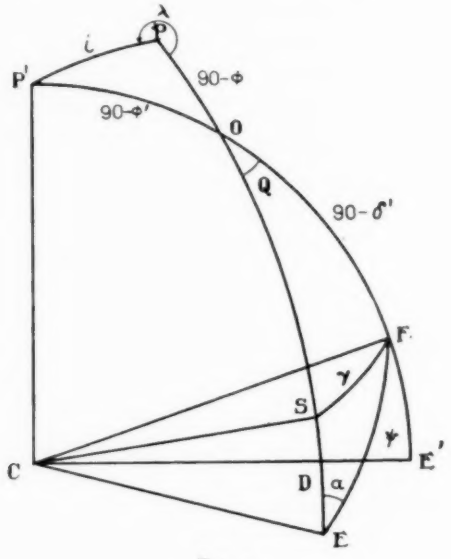


FIG. 1

γ = SF = angle between line of sight and line of force at observed point.
i = PP' = inclination of magnetic axis to solar axis of rotation.
λ = $\angle OPP'$ = longitude of magnetic pole referred to central meridian and measured in direction of sun's rotation.
ψ = $\angle ECF$ = arc EF.
α = $\angle SEF$.

The solution of the problem requires the expression of $H'\cos\gamma$ as a function of ϕ, D, λ, and i. From $\triangle ESF$

$$\cos\gamma = \cos D\cos\psi + \sin D \sin\psi\cos\alpha\,. \qquad (22)$$

From $\triangle EOF$

$$\left.\begin{aligned}\cos\psi &= \cos\phi \sin\delta' + \sin\phi \cos\delta' \cos Q \\ \sin\psi \cos\alpha &= \sin\phi \sin\delta' - \cos\phi \cos\delta' \cos Q\end{aligned}\right\} \tag{23}$$

The inclination of the lines of force to the sun's surface at the observed point is connected with the magnetic latitude by the relation[1]

$$\tan\delta' = 2\tan\phi' \tag{24}$$

whence we find

$$H'\sin\delta' = 2\sin\phi', \quad H'\cos\delta' = \cos\phi' \tag{25}$$

in which H' has the value defined by (20).

Substituting (25) into (23)

$$\left.\begin{aligned}H'\cos\psi &= 2\cos\phi \sin\phi' + \sin\phi \cos\phi' \cos Q \\ H'\sin\psi \cos\alpha &= 2\sin\phi \sin\phi' - \cos\phi \cos\phi' \cos Q\end{aligned}\right\} \tag{26}$$

From $\triangle OPP'$

$$\left.\begin{aligned}\sin\phi' &= L\sin\phi + M\cos\phi \\ \cos\phi'\cos Q &= L\cos\phi - M\sin\phi\end{aligned}\right\} \tag{27}$$

in which

$$L = \cos i, \quad M = \sin i \cos\lambda. \tag{28}$$

The substitution of (27) into (26) gives

$$\left.\begin{aligned}2H'\cos\psi &= 3L\sin 2\phi + 3M\cos 2\phi + M \\ 2H'\sin\psi\cos\alpha &= 3M\sin 2\phi - 3L\cos 2\phi + L\end{aligned}\right\} \tag{29}$$

Then, from the substitution of (29) into (23), we have

$$2H'\cos\gamma = 3L\sin(2\phi - D) + 3M\cos(2\phi - D) + L\sin D + M\cos D. \tag{30}$$

Combining (21) and (30) and restoring the values of L and M,

$$\left.\begin{aligned}k\Delta = \tfrac{1}{2}3\sin(2\phi - D) + \sin D\{\cos i + \\ \tfrac{1}{2}3\cos(2\phi - D) + \cos D\{\sin i\cos\lambda\end{aligned}\right\} \tag{31}$$

which is the required equation. The factor k is a new constant depending upon the units and the field-strength at the sun's magnetic equator.

Since the coefficients of $\cos i$ and $\sin i\cos\lambda$ in (31) may be replaced by expressions of the form $n\cos N$ and $n\sin N$ it appears that, whatever the values of D, i, and λ, the displacements always define a sine curve. But the latitude of the sun's center, D, is small, never exceeding 7°, and the observations show that i is also

[1] Foster and Porter, *loc. cit.*

small, if not actually zero. The displacement-curve is therefore approximately $3 \sin 2\phi$. Its ordinates are zero near the equator and the poles; they have opposite signs in the two hemispheres, and maximum absolute values near $\phi = 45°$, north and south.

Equation (31) contains three unknowns, k, i, and λ. For their determination, consider first values of the displacements for numerically equal northern and southern latitudes, observed on or near the same date. The angle D will then be sensibly unchanged. Denoting the displacements by Δ_n and Δ_s, we find

$$k(\Delta_n - \Delta_s) = 6 \sin 2\phi \, (\cos D \cos i + \sin D \sin i \cos \lambda) . \qquad (32)$$

The parenthesis in the right member of this expression differs from unity by a quantity of the second order in D and i, and it is quite sufficient to write

$$k = \frac{6 \sin 2\phi}{\Delta_n - \Delta_s} . \qquad (33)$$

The denominator has its maximum near $\phi = 45°$, and the equation is most advantageously used for points near this latitude.

Writing now (31) in the form

$$k\Delta = A \cos i + B \sin i \cos \lambda , \qquad (34)$$

we have, with sufficient approximation,

$$\sin i \cos \lambda = \frac{k\Delta - A}{B} . \qquad (35)$$

The application of this expression should be restricted to points near the equator. If the values of $\sin i \cos \lambda$ derived from a series of observations extending over one or more revolutions of the sun be plotted with values of the time as abscissae, they will define a curve whose amplitude will be $\sin i$. The intersections of the curve with the axis will indicate the epochs when the longitude of the magnetic pole referred to the central meridian is 90° or 270°.

For the application of (35) it will be convenient to tabulate the values of A and B with the arguments ϕ and D; owing to the limitation of the equation to latitudes near the equator, the tables need not be extensive.

Equations (33) and (35) serve for the determination of k, i, and λ. Although the introduction of k affords a convenient arrangement of the formulae for the calculation of i and λ, we are really

interested in the value of the solar field-strength H_e upon which k depends.

By comparing (18) and (21), it appears that when Δ is expressed in Ångström units, c represents the separation in Ångströms of an n-component from the p-component of the line observed produced by a field of 1 gauss. When this quantity has been determined by appropriate laboratory investigations, the solar field-strength can be determined. For (21) may be written

$$\Delta = cH_eF \tag{36}$$

in which Δ is now supposed to be expressed in Ångströms, while F represents the right-hand member of (31). When i and λ have been found by the method outlined, H_e may be calculated by (36), or by an equation analogous to (32), namely

$$\Delta_n - \Delta_s = 3cH_p \sin 2\phi(\cos D \cos i + \sin D \sin i \cos \lambda), \tag{37}$$

into which the polar field-strength $H_p = 2H_e$ has been introduced. For a first approximation we may write $i = 0$, $\cos D = 1$; and applying (37) to $\phi = 45°$, we have for the polar field-strength

$$H_p = \frac{2\Delta_{45}}{3c} \tag{38}$$

in which Δ_{45} is the mean displacement at 45°.

Thus far it has been assumed that the line observed is a normal Zeeman triplet. It is obvious, however, that the applicability of the formulae is not restricted to lines of this character. For the n- and p-components of the normal triplet, we may substitute, throughout, the groups of n- and p-components of lines of more complicated structure. Certain conditions must, however, be fulfilled. For example, the total intensity of the n-components, when examined perpendicularly to the field, must equal that of the p-components. Further, the various groups of components considered as blended lines must possess an intensity distribution which is symmetrical with respect to the centers of the groups. Otherwise the preceding developments presuppose no conditions which in practice would not be fulfilled. It is impossible to indicate the percentage of lines satisfying these conditions, but probably the number is considerable.

If in any given case there is reason to suppose that one or more

of the above conditions is not fulfilled, it will still be possible to determine the strength of the solar field. It is only necessary to consider the displacement at that point of the solar surface for which the lines of the force are parallel to the line of sight. There the light received in the spectrum comes wholly from one of the n-components, and the relative displacement in adjacent strips is directly the Zeeman separation of the line. Since neither the p-component nor the second n-component enters, there is no question of relative intensities or of a resultant blend, and we are independent of the structure of the line. The condition of parallelism means that the points S and F in Fig. 1 must coincide, which can occur only when $\lambda = 0^\circ$ or 180°; otherwise both S and F would have to coincide with O, which is an impossible condition, for $ES = D$ is restricted to values less than 7°, while F can coincide with O only when O is at P'. Since, however, i is certainly small, we may substitute for the coincidence of S and F the condition $OS = OF$ or

$$90^\circ - \delta' = \phi - D ,$$

which will never be greatly in error, and indeed will be exact when $\lambda = 0^\circ$ or 180°. By (24) this becomes

$$2 \tan \phi' = \cot(\phi - D) . \tag{39}$$

As an approximation we may write $\phi' = \phi$, $D = 0$, whence

$$\tan^2 \phi = \tfrac{1}{2} \text{ or } \phi = 35^\circ.3 .$$

Applying therefore equation (21) to this latitude we have simply

$$\Delta_{35} = 2cH_e H'$$

from which, with the aid of (20)

$$H_p = \frac{\Delta_{35}}{\sqrt{2}c} \tag{40}$$

The preceding developments are complete in so far as they relate to the transformations taking place in the beam of light incident upon the quarter-wave plate. They disregard, however, the fact that the beam has undergone two reflections from the silvered surfaces of the coelostat mirrors before reaching the polarizing apparatus. These reflections modify the polarization produced in the solar lines by the sun's general field and presumably influence to some degree the observed displacements. Although Mr. Hale has

shown, by means of the polarimeter,[1] that under normal observing conditions the elliptical polarization introduced by the mirrors is small, it is of interest to consider the matter analytically. From the standpoint of a definite numerical determination, such an investigation cannot be regarded as complete, owing to the lack of specific information as to the optical constants of the reflecting surfaces. These probably vary within rather wide limits with increasing age of the silver coat. Nevertheless, the results for a normal silver surface should afford important indications as to the magnitude of the influence upon the displacements of the solar lines, and its variation with the changing conditions of observation.

Before proceeding to an examination of the modifications imposed upon the light-vibrations by the successive reflections and their subsequent passage through the polarizing apparatus, we shall consider the general results of metallic reflection. These are expressed by the equation[2]

$$\frac{R_p}{R_n} e^{i\Delta} = -\frac{I_p}{I_n} \frac{\cos(i+r)}{\cos(i-r)} \tag{41}$$

in which I and R represent the amplitudes of the incident and reflected vibrations. The subscripts p and n refer to directions respectively parallel to and perpendicular to the plane of incidence, and perpendicular to the direction of the incident or reflected ray, as the case may be. Δ is the excess of the change in phase produced by the reflection of the p-component over that of the n-component (not to be confused with the displacement Δ used above). In the left member i indicates the imaginary radical, while in the right, it is the angle of incidence. The angle r, analogous to the angle of refraction in the case of a transparent medium, must be replaced by the complex dielectric constant, which in turn is a function of the optical constants of the reflecting surface. Writing

$$\rho' = -e^{-i\Delta} \frac{\cos(i+r)}{\cos(i-r)}, \tag{42}$$

[1] *Contributions from Mount Wilson Solar Observatory*, No. 71; *Astrophysical Journal*, July, 1913.

[2] Drude, "Theory of Optics", translated by Mann and Millikan, 282, 361, 1902.

(41) may be replaced by the two equations

$$\left.\begin{aligned} R_p &= hI_p\rho' \\ R_n &= hI_n \end{aligned}\right\} \tag{43}$$

which are the relations connecting the incident and reflected amplitudes parallel to and perpendicular to the plane of incidence. Concerning the factor h we need not inquire further at the moment.

From (42) we may derive[1]

$$\frac{1+\rho' e^{i\Delta}}{1-\rho' e^{i\Delta}} = \cot Pe^{iQ}. \tag{44}$$

P and Q are defined by

$$\left.\begin{aligned} \tan P &= \frac{n\sqrt{1+\kappa^2}}{\sin i \tan i} \\ \tan Q &= \kappa \end{aligned}\right\} \tag{45}$$

which connect ρ' and Δ with the optical constants n and κ.

Replacing in (44) the exponentials by the corresponding trigonometric functions and equating the real and imaginary parts of the resulting expression we find

$$\left.\begin{aligned} \rho' \sin \Delta &= \frac{\sin Q \sin 2P}{1+\cos Q \sin 2P} \\ \rho' \cos \Delta &= \frac{\cos 2P}{1+\cos Q \sin 2P} \end{aligned}\right\} \tag{46}$$

whence

$$\left.\begin{aligned} \tan \Delta &= \sin Q \tan 2P \\ \rho'^2 &= \frac{1-\cos Q \sin 2P}{1+\cos Q \sin 2P} \end{aligned}\right\} \tag{47}$$

By means of (45) the latter of these may be written

$$\rho'^2 = \frac{(\sin i \tan i - n)^2 + n^2\kappa^2}{(\sin i \tan i + n)^2 + n^2\kappa^2}. \tag{48}$$

An examination of (48) shows that for $i = 0°$ or $90°$, $\rho'^2 = 1$; and that ρ'^2 has a minimum value of

$$\rho'^2 = \frac{\sqrt{1+\kappa^2}-1}{\sqrt{1+\kappa^2}+1}$$

[1] Drude, *op. cit.* pp. 361–64.

when

$$\sin i \tan i = n\sqrt{1+\kappa^2}, \tag{49}$$

that is, when i is equal to the principal angle of incidence. For all other values of i, ρ'^2 lies between these two limits. For normal silver and for sodium light the extinction coefficient κ, and the index n, have the values[1]

$$\kappa = 20, \quad n = 0.18.$$

The minimum value of ρ'^2 is therefore 0.9, and of ρ', 0.95. But these are attained only for the polarizing angle, which for silver, as may be found from (49), is approximately 75°. In practice, the angles of incidence will be much smaller than the polarizing angle and ρ' will differ but little from unity.

For the purposes of this investigation we may therefore replace (43) by

$$\left.\begin{aligned} R_p &= hI_p \\ R_n &= hI_n \end{aligned}\right\} \tag{50}$$

which will notably simplify the later discussion. Further, it appears from (45) that Q is approximately 87°. In consequence, we write $\sin Q = 1$, and find from (45) and (47)

$$\tan \tfrac{1}{2}\Delta = \frac{3.67}{\sin i \tan i} \tag{51}$$

Equations (50) and (51) are the relations defining the phenomena at the reflecting surfaces. We now turn to the application of these expressions to the problem under consideration.

In accordance with the previous notation the polarized vibrations incident upon the coelostat mirrors are of the form

$$\left.\begin{aligned} x &= a \sin \omega_1 t, & x &= 0 & x &= a \sin \omega_2 t \\ y &= b \cos \omega_1 t, & y &= \sqrt{2}a \sin \gamma \cos \omega t, & y &= -b \cos \omega_2 t \end{aligned}\right\} \tag{52}$$

in which $b = a \cos \gamma$; γ, as before, denotes the inclination of the line of sight to the lines of force cutting the solar surface at the point of observation. The first and last pair of equations in (52) represent the opposite circular vibrations of the outer components of the triplet, and the second pair the linear vibrations of the middle component. The axes to which (52) are referred are perpendicular

[1] Drude, *op cit.* p. 366.

to the first incident beam, with OY oriented in the direction of the central solar meridian.

These vibrations must now be followed through the successive reflections. It will be sufficient to develop the formulae for the first, as the results for the others can immediately be found by an appropriate change in the amplitude. For brevity the distinguishing subscript for the frequency ω will be omitted.

The course of the reflected beam is indicated by Fig. 2, which is in the form of a projection of the celestial sphere on the plane of the horizon. C is the center of the sphere; CMS', the meridian; P, the north pole; WME, the celestial equator; and S, the sun whose position is defined by $PS=90°-\delta$ and the hour angle $MPS=t$. The normal to the coelostat mirror at C lies in the plane of the equator and cuts the sphere at N. The first reflected ray has the direction CP', such that the angle of incidence $NS=i=NP'$. It follows at once that the polar distance of P' is $90°+\delta$. Its hour angle we will call t'. The direction CP' is also that of the center of the second mirror as seen from the center of the first, and may be defined by the co-ordinates A and z, the azimuth and zenith distance of the point P'.

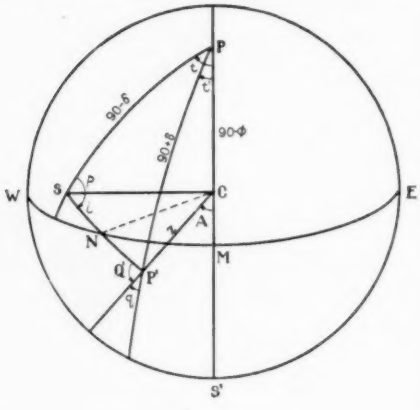


FIG. 2

The course of the beam of light is from S to C, then from C in the direction CP' until it strikes the second flat, whence it is reflected vertically downward through the objective of the telescope, reaching finally the polarizing apparatus and the spectrograph. The first incidence plane is therefore SCP'; the second, the vertical plane through C and P'. The angle between the two incidence planes is denoted by Q'. The first angle of incidence is i and the second $\frac{1}{2}z$.

We shall denote co-ordinates parallel to and perpendicular to

the first incidence plane (both perpendicular to the incident or the reflected ray, as the case may be) by p and n, respectively, and similar co-ordinates referred to the second incidence plane by p' and n'. The subscripts i and r will differentiate the incident and reflected rays. The convention governing the positive direction of p and n is as follows: Look along the incident (or reflected) ray in the direction of the reflecting surface with the normal directed upward. The positive direction of p will then be upward, and of n, to the left.

Figs. 3, 4, and 5 show the positive directions of the various sets of reference axes as seen from without the celestial sphere. The

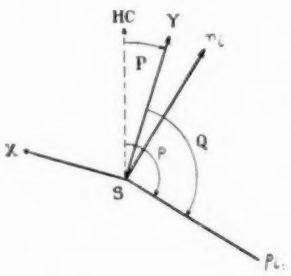


FIG. 3

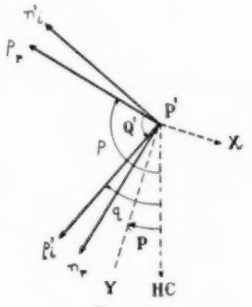


FIG. 4

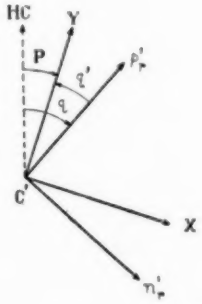


FIG. 5

general configuration is the same as Fig. 2, the north pole (actual, not reflected) in each instance lying above the figure in the direction of the hour circle HC. Fig. 3 gives the orientation in looking along the line SC in Fig. 2; Fig. 4, that seen in the direction $P'C$; and Fig. 5 that for the final direction of the beam (vertically downward) after reflection from the second flat whose center is at C' and situated on the line $P'C$. Thus p_i and p_r (Figs. 3, 4) lie in the plane SCP' and are respectively perpendicular to SC and $P'C$. Further, p'_i lies in the vertical plane through P' and C and is perpendicular to $P'C$. The direction of Y, the central solar meridian, is determined by the position angle of the sun's north point, P. From Fig. 3

$$Q = p - P. \tag{53}$$

The hour circle HC in Fig. 4 is the reflection of HC in Fig. 3. Since this coincides in direction with the actual hour circle $P'P$ (Fig. 2) through P', the relations between the angles p, q, Q' are as shown in Fig. 4, namely

$$Q'=p-q\,. \tag{54}$$

The final direction of the central meridian must be determined, as that fixes the position of the slit. It is indicated in Fig. 5 by $C'Y$. The orientation is given by

$$q'=q-P\,. \tag{55}$$

With these preliminary definitions established we may proceed to the detailed consideration of (52).

The determination of the resultant intensities parallel to and perpendicular to the slit involves the following steps: (*a*) a rotation of the reference axes XY through the angle Q, which gives the values of n_i and p_i; (*b*) an application of the reflection equations (50) and (51) giving n_r and p_r; (*c*) a rotation of the axes through the angle Q', which determines n_i' and p_i'; (*d*) a second application of the reflection equations resulting in n_r' and p_r'; (*e*) a rotation of the axes through the angle q', giving the co-ordinates x_i and y_i of the vibrations incident upon the quarter-wave plate, referred to directions respectively perpendicular to and parallel to the slit; (*f*) the introduction of the transformation produced by the quarter-wave plate, which determines the co-ordinates x' and y' of the vibrations falling upon the Nicol. The squares of the amplitudes of the resultant vibrations parallel to x' and y' are the required intensities. These steps will be considered in turn.

a) The typical elliptic vibration

$$x=a\sin\omega t\,,\quad y=b\cos\omega t \tag{56}$$

may be written

$$\left.\begin{aligned} x&=A\sin\omega t+B\sin\omega t\\ y&=A\cos\omega t-B\cos\omega t\end{aligned}\right\} \tag{57}$$

in which

$$A=\tfrac{1}{2}(a+b),\quad B=\tfrac{1}{2}(a-b)\,. \tag{58}$$

Equations (57) represent two superimposed circular vibrations, the first of which is negative (clockwise), the second positive. The rotation of the axes through the negative angle Q (Fig. 3) decreases

the phase of the first circular vibration, and increases that of the second, by Q. Consequently

$$\left.\begin{aligned} n_i &= A \sin(\omega t - Q) + B \sin(\omega t + Q) \\ p_i &= A \cos(\omega t - Q) - B \cos(\omega t + Q) \end{aligned}\right\} \tag{59}$$

b) The application of the reflection equations (50) and (51) requires the multiplication of the incident amplitudes A and B by the factor h, and the introduction of the relative phase change Δ in n and p. As a result of the former, we shall have h appearing as a common factor in all of the amplitudes. Since we are interested only in relative intensities, we may omit this factor and assume that the reflected amplitudes are directly A and B. The changes of phase in n and p are respectively δ_p and δ_n, with the condition

$$\delta_p - \delta_n = \Delta .$$

We shall suppose

$$\delta_p = \delta + \tfrac{1}{2}\Delta , \quad \delta_n = \delta - \tfrac{1}{2}\Delta .$$

Since the origin of phase is arbitrary, the common part δ may be dropped. The co-ordinates of the vibrations reflected from the first mirror are therefore

$$\left.\begin{aligned} n_r &= A \sin\left(\omega t - Q - \frac{\Delta}{2}\right) + B \sin\left(\omega t + Q - \frac{\Delta}{2}\right) \\ p_r &= A \cos\left(\omega t - Q + \frac{\Delta}{2}\right) - B \cos\left(\omega t + Q + \frac{\Delta}{2}\right) \end{aligned}\right\} \tag{60}$$

Expanding, (60) may be written

$$\left.\begin{aligned} n_r = {} & A \cos\frac{\Delta}{2} \sin(\omega t - Q) - A \sin\frac{\Delta}{2} \cos(\omega t - Q) + \\ & B \cos\frac{\Delta}{2} \sin(\omega t + Q) - B \sin\frac{\Delta}{2} \cos(\omega t + Q) \\ p_r = {} & A \cos\frac{\Delta}{2} \cos(\omega t - Q) - A \sin\frac{\Delta}{2} \sin(\omega t - Q) - \\ & B \cos\frac{\Delta}{2} \cos(\omega t + Q) + B \sin\frac{\Delta}{2} \sin(\omega t + Q) \end{aligned}\right\} \tag{61}$$

c) Equations (61) represent four superimposed circular vibrations whose amplitudes are $A \cos\frac{\Delta}{2}$, $A \sin\frac{\Delta}{2}$, $B \cos\frac{\Delta}{2}$, and $B \sin\frac{\Delta}{2}$. The first and fourth are negative, the second and third positive.

The rotation of the axes through the positive angle Q' (Fig. 4) for the purpose of obtaining the co-ordinates n'_i and p'_i perpendicular to and parallel to the second incidence plane decreases the phase of the positive circular vibrations, and increases that of the negative vibrations, by Q'. It will be observed from Fig. 4 that such a rotation will bring p_r into coincidence with p'_i, but that n_r will be opposite in direction to n'_i. Consequently the sign of the first co-ordinate must be changed. We have therefore from (61)

$$\begin{aligned} n'_i = &-A\cos\frac{\Delta}{2}\sin(\omega t-Q+Q')+A\sin\frac{\Delta}{2}\cos(\omega t-Q-Q') \\ &-B\cos\frac{\Delta}{2}\sin(\omega t+Q-Q')+B\sin\frac{\Delta}{2}\cos(\omega t+Q+Q') \\ p'_i = &+A\cos\frac{\Delta}{2}\cos(\omega t-Q+Q')-A\sin\frac{\Delta}{2}\sin(\omega t-Q-Q') \\ &-B\cos\frac{\Delta}{2}\cos(\omega t+Q-Q')+B\sin\frac{\Delta}{2}\sin(\omega t+Q+Q') . \end{aligned}$$

Expanding the sine and cosine factors and restoring the values of a and b by (58), it appears that these expressions may be written in the form

$$\left.\begin{aligned} n'_i &= A_n\sin\omega t+B_n\cos\omega t \\ p'_i &= A_p\sin\omega t+B_p\cos\omega t \end{aligned}\right\} \tag{62}$$

in which

$$\left.\begin{aligned} A_n &= -a\cos\frac{\Delta}{2}\cos(Q-Q')+b\sin\frac{\Delta}{2}\sin(Q+Q') \\ B_n &= +b\cos\frac{\Delta}{2}\sin(Q-Q')+a\sin\frac{\Delta}{2}\cos(Q+Q') \\ A_p &= +a\cos\frac{\Delta}{2}\sin(Q-Q')-b\sin\frac{\Delta}{2}\cos(Q+Q') \\ B_p &= +b\cos\frac{\Delta}{2}\cos(Q-Q')+a\sin\frac{\Delta}{2}\sin(Q+Q') \end{aligned}\right\} \tag{63}$$

d) The second application of the reflection equations is precisely similar to the first. Denoting the relative change of phase by Δ', we have from (62)

$$\left.\begin{aligned} n'_r &= A_n\sin\left(\omega t-\frac{\Delta'}{2}\right)+B_n\cos\left(\omega t-\frac{\Delta'}{2}\right) \\ p'_r &= A_p\sin\left(\omega t+\frac{\Delta'}{2}\right)+B_p\cos\left(\omega t+\frac{\Delta'}{2}\right) \end{aligned}\right\} \tag{64}$$

These may be written

$$\left.\begin{aligned} n'_r &= A'_n \sin \omega t + B'_n \cos \omega t \\ p'_r &= A'_p \sin \omega t + B'_p \cos \omega t \end{aligned}\right\} \tag{65}$$

in which

$$\left.\begin{aligned} A'_n &= A_n \cos \frac{\Delta'}{2} + B_n \sin \frac{\Delta'}{2}\,, \quad B'_n = -A_n \sin \frac{\Delta'}{2} + B_n \cos \frac{\Delta'}{2} \\ A'_p &= A_p \cos \frac{\Delta'}{2} - B_p \sin \frac{\Delta'}{2}\,, \quad B'_p = +A_p \sin \frac{\Delta'}{2} + B_p \cos \frac{\Delta'}{2} \end{aligned}\right\} \tag{66}$$

e) The values of the co-ordinates perpendicular to and parallel to the slit are connected (Fig. 5) with n'_r and p'_r by

$$\begin{aligned} x_i &= n'_r \cos q' + p'_r \sin q' \\ y_i &= -n'_r \sin q' + p'_r \cos q'\,. \end{aligned}$$

Substituting from (65), we find

$$\left.\begin{aligned} x_i &= \alpha \sin \omega t + \beta \cos \omega t \\ y_i &= \gamma \sin \omega t + \delta \cos \omega t \end{aligned}\right\} \tag{67}$$

in which

$$\left.\begin{aligned} \alpha &= +A'_n \cos q' + A'_p \sin q'\,, \quad \beta = +B'_n \cos q' + B'_p \sin q' \\ \gamma &= -A'_n \sin q' + A'_p \cos q'\,, \quad \delta = -B'_n \sin q' + B'_p \cos q' \end{aligned}\right\} \tag{68}$$

f) To introduce the transformation effected by the quarter-wave plate we require the components of (67) in the direction of the principal axes of the mica strips composing the plate. For two adjacent strips these are inclined to the axes of x_i and y_i at angles of $+45°$ and $-45°$. We need consider but the first, for which

$$\left.\begin{aligned} \xi &= \frac{1}{\sqrt{2}}(+x_i + y_i) = \frac{1}{\sqrt{2}}(\gamma+\alpha) \sin \omega t + \frac{1}{\sqrt{2}}(\delta+\beta) \cos \omega t \\ \eta &= \frac{1}{\sqrt{2}}(-x_i + y_i) = \frac{1}{\sqrt{2}}(\gamma-\alpha) \sin \omega t + \frac{1}{\sqrt{2}}(\delta-\beta) \cos \omega t \end{aligned}\right\} \tag{69}$$

Assuming that the "fast" direction of the mica strip considered coincides with ξ, and denoting the emergent values of ξ and η by ξ' and η', we have

$$\left.\begin{aligned} \xi' &= \frac{1}{\sqrt{2}}(\gamma+\alpha) \cos \omega t - \frac{1}{\sqrt{2}}(\delta+\beta) \sin \omega t \\ \eta' &= \frac{1}{\sqrt{2}}(\gamma-\alpha) \sin \omega t + \frac{1}{\sqrt{2}}(\delta-\beta) \cos \omega t \end{aligned}\right\} \tag{70}$$

in which the common part of the phase change in ξ and η is neglected. The components of (70) perpendicular to and parallel to the slit are given by

$$\left.\begin{aligned} x' &= \frac{1}{\sqrt{2}}(\xi'-\eta') = a' \sin \omega t + \beta' \cos \omega t \\ y' &= \frac{1}{\sqrt{2}}(\xi'+\eta') = \gamma' \sin \omega t + \delta' \cos \omega t \end{aligned}\right\} \tag{71}$$

in which

$$\left.\begin{aligned} a' &= \tfrac{1}{2}(+a-\beta-\gamma-\delta), \quad \beta' = \tfrac{1}{2}(a+\beta+\gamma-\delta) \\ \gamma' &= \tfrac{1}{2}(-a-\beta+\gamma-\delta), \quad \delta' = \tfrac{1}{2}(a-\beta+\gamma+\delta) \end{aligned}\right\} \tag{72}$$

Since the two terms in the right members of (71) differ in phase by 90°, the intensities of the vibrations incident upon the Nicol parallel to x' and y' are respectively

$$I_x = a'^2+\beta'^2, \quad I_y = \gamma'^2+\delta'^2.$$

For an adjacent strip of the quarter-wave plate the results will be the same, but with an interchange of the values of I_x and I_y. As the Nicol transmits only those vibrations which are parallel to the slit, we have for the intensity of the light entering the spectrograph from the two strips

$$I = a'^2+\beta'^2, \quad I' = \gamma'^2+\delta'^2. \tag{73}$$

The expression of I and I' in terms of the original amplitude a, the phase retardations Δ and Δ', and the various angles involved is accomplished by successive substitution back through (73), (72), (68), (66), and (63). The operations involved are tedious, but not difficult. Only the results of the successive steps need be given here. From (73) and (72) we obtain

$$\left.\begin{aligned} I &= \tfrac{1}{2}(a^2+\beta^2+\gamma^2+\delta^2) - a\delta + \beta\gamma \\ I' &= \tfrac{1}{2}(a^2+\beta^2+\gamma^2+\delta^2) + a\delta - \beta\gamma \end{aligned}\right\} \tag{74}$$

By (68), (66), and (63),

$$\begin{aligned} a^2+\beta^2+\gamma^2+\delta^2 &= A'^2_n + A'^2_p + B'^2_n + B'^2_p \\ &= A_n^2 + A_p^2 + B_n^2 + B_p^2 \\ &= a^2+b^2. \end{aligned} \tag{75}$$

Again, by (68) and (66),

$$\begin{aligned} -a\delta+\beta\gamma &= A'_pB'_n - A'_nB'_p \\ &= -(A_nA_p+B_nB_p)\sin\Delta' + (A_pB_n-A_nB_p)\cos\Delta'. \end{aligned} \tag{76}$$

Further, by (63),

$$A_n A_p + B_n B_p = \tfrac{1}{2}(a^2 - b^2)(\cos 2Q \sin 2Q' - \sin 2Q \cos 2Q' \cos \Delta) + ab \cos 2Q' \sin \Delta . \tag{77}$$

$$A_p B_n - A_n B_p = \tfrac{1}{2}(a^2 - b^2) \sin 2Q \sin \Delta + ab \cos \Delta . \tag{78}$$

Substituting (77) and (78) into (76), and the result, together with (75), into (74) gives

$$\left.\begin{aligned} I &= \tfrac{1}{2}(a^2+b^2)+\lambda+\mu \\ I' &= \tfrac{1}{2}(a^2+b^2)-\lambda-\mu \end{aligned}\right\} \tag{79}$$

in which λ and μ have the values

$$\left.\begin{aligned} \lambda &= ab(\cos \Delta \cos \Delta' - \sin \Delta \sin \Delta' \cos 2Q') \\ \mu &= \tfrac{1}{2}(a^2-b^2)\{\sin 2Q \sin \Delta \cos \Delta' - \\ &\quad (\cos 2Q \sin 2Q' - \sin 2Q \cos 2Q' \cos \Delta) \sin \Delta'\} \end{aligned}\right\} \tag{80}$$

The original elliptic vibration (56) whose intensities are given by (79) has the form of the first of (52). Let us assume that this represents the red component of the triplet line observed. The intensities of the violet component may be found by changing the sign of b in (79) and (80). Further, the intensities for the middle component may be derived by writing $a=0$, $b=1/\sqrt{2}\, a \sin \gamma$. Since for the red and violet components, b in (80) has the value $a \cos \gamma$, we have

$$\begin{aligned} \lambda_V &= -\lambda_R , \quad \mu_V = \mu_R \\ \lambda_M &= 0 , \quad \mu_M = -2\mu_R = -2\mu_V \end{aligned}$$

in which the subscripts designate the components to which the values of λ and μ refer. Denoting now the intensities of the three components in one of two adjacent spectrum strips by V, M, and R, we have from (79)

$$\left.\begin{aligned} V &= \tfrac{1}{2}a^2(1+\cos^2\gamma)-\lambda+\mu \\ M &= a^2\sin^2\gamma \qquad\qquad -2\mu \\ R &= \tfrac{1}{2}a^2(1+\cos^2\gamma)+\lambda+\mu \end{aligned}\right\} \tag{81}$$

The intensities V', M', R', for the second spectrum strip are the same as (81) except that the signs of λ and μ are reversed.

Equations (81) correspond to (10) of the first part of the development. For the determination of the displacement of the resultant

of the three components we proceed as in the case of equations (12)-(18). Since (81), by,

$$R-V=2\lambda\,,\quad V+M+R=2a^2\,.$$

we have finally

$$\Delta=2\,\sigma\epsilon\,\cos\,\gamma, \tag{82}$$

in which the factor ϵ has the value

$$\epsilon=\cos\,\Delta_1\,\cos\,\Delta_2-\sin\,\Delta_1\,\sin\,\Delta_2\,\cos\,2Q'\,. \tag{83}$$

The symbols Δ and Δ' previously used for the relative phase retardations at the reflecting surfaces are here replaced by Δ_1 and Δ_2 to avoid a possible confusion with the displacement Δ in (82). If (82) be compared with (18), it will be seen that the effect of the elliptical polarization produced by the two reflections from the coelostat mirrors is only to introduce the factor ϵ defined by (83). Q', it will be recalled, is the angle included between the two incidence planes.

The angles Δ_1, Δ_2, and Q' are all functions of t and δ, the hour angle and declination of the sun. Although ϵ is not readily expressed explicitly in terms of these quantities, its values are easily calculated for any given t and δ. To obtain an idea of the magnitude of the polarization effect, it will be convenient to tabulate ϵ with t and δ as arguments.

For the calculation of the phase retardations, the angles of incidence are required, and for Q' the angles p and q. From $\triangle SPN$, Fig. 2 we find

$$\tan\,p=-\tan\,\tau\,\operatorname{cosec}\,\delta\,,\quad \sin\,i=\sin\,\tau\,\operatorname{cosec}\,p, \tag{84}$$

in which

$$\tau=\tfrac{1}{2}(t-t')\,.$$

Further from $\triangle PCP'$,

$$\sin\,q\,\sin\,z=\cos\,\phi\,\sin\,t'\,. \tag{85}$$

The angle of incidence for the second mirror is $i'=\frac{1}{2}z$. These relations may be used for the calculations of i, i', and $Q'=p-q$, whence Δ_1 and Δ_2 may be derived by (51). There remains still, however, the expression of z and t' as functions of δ. These quantities are the zenith distance and hour angle of the center of the second mirror as seen from the center of the first, and are independent of t.

The elevation of the second flat above the first is 915 mm. For morning observations the coelostat is set 600 mm east of the second mirror, while during the afternoon it is 600 mm west. This increases the average angle of incidence for the first mirror, but permits a continuation of observations past the meridian without interference from the shadow of the second flat.

If d represent the projection upon a horizontal plane of the line joining the centers of the two mirrors, and if A and z be the azimuth and zenith distance of the second referred to the first, then

$$\sin A = \frac{600}{d}, \quad \tan z = \frac{d}{915},$$

whence

$$\sin A \tan z = \frac{600}{915}. \tag{86}$$

Now from $\triangle PCP'$, Fig. 2

$$\sin A \sin z = \cos \delta \sin t',$$

and by (86)

$$\cos z = 1.525 \cos \delta \sin t'. \tag{87}$$

Again, from the same triangle

$$\cos z = -\sin \delta \sin \phi + \cos \delta \cos \phi \cos t'. \tag{88}$$

It is to be noted that for the conditions here assumed, the values of t' and q are negative. The last two equations serve for the determination of z and t' in terms of δ. The solution is therefore complete, the necessary formulae for the derivation of ϵ being (87) and (88), which are readily solved by approximations, and (85), (84), (54), (51), and (83).

Their application gives the following results:

VALUES OF THE POLARIZATION FACTOR ϵ

δ	t						
	0^h	1^h	2^h	3^h	4^h	5^h	6^h
$+25°$	0.96	0.95	0.94	0.93	0.91	0.86	0.78
$+15$	0.98	0.98	0.97	0.96	0.94	0.88	0.78
$+5$	0.99	0.99	0.99	0.97	0.95	0.88	0.77
-5	1.00	1.00	0.99	0.97	0.93	0.86	0.72
-15	1.00	0.99	0.98	0.95	0.90	0.81	0.63
-25	0.99	0.98	0.96	0.92	0.84	0.72	0.51

The values for the optical constants used in (51) are for normal silver and relate to sodium light, which agrees closely in wave-length with the lines observed by Mr. Hale. It cannot be supposed, however, that the values used will apply accurately to the mirrors, even when freshly silvered and burnished. Drude has found,[1] for example, that but few metals remain uncontaminated when polished with rouge, and the account of his experiments seems to imply that this is the case with silver. Generally speaking, variations in the constants may be produced by the presence of a superficial film of some foreign substance, and by minute scratches or furrows in the surface of the metal. Although the results for silver are extremely sensitive to the latter, it is probable that with a carefully burnished mirror we may disregard this source of disturbance. As for the former, Drude has shown[2] that the presence of a superficial film is invariably to reduce the principal angle of incidence, and also the value of Δ corresponding to any given incidence, the amount depending upon the thickness and the index of refraction of the film. The principal azimuth, on the other hand is slightly increased.

These modifications affect the above results in two directions: first, through the approximations used for ρ' and Q, whose true values are defined by (45) and (48), and second, by a direct change in ϵ. Since

$$\kappa = \tan Q = \tan 2\bar{\psi}, \qquad (2\bar{\psi} < 90^\circ)$$

in which $\bar{\psi}$ is the principal azimuth,[3] it follows that the minimum value of ρ' (p. 15) is increased by the presence of the superficial film, and that the approximation for Q is likewise improved. The effect upon ϵ, however, is less favorable. The form of (51) assumes that for values of i less than the principal incidence, which corresponds to the conditions of observation, $180^\circ > \Delta > 90^\circ$. A decrease in Δ therefore means a decrease in ϵ; but beyond this general statement it is impossible to specify without definite knowledge of the properties of the film. Broadly speaking, however, it may be said that, with ordinary care in the treatment of the mirrors, it is very improbable that any serious modification

[1] *Annalen der Physik*, **39**, 503, 1890. [2] *Ibid.*, p. 488.

[3] Drude, "Theory of Optics," translated by Mann and Millikan, p. 364, 1902.

of the results will occur, provided that the observations be restricted to moderate values of the hour angle.

The preceding results are in part a continuation of an investigation begun by Mr. J. A. Anderson, who first derived by a somewhat different method equations (10) for the intensities of the three components in any spectrum strip. These were numerically applied by him to a special case, in which it was assumed that the observer is in the plane of the sun's equator and that the magnetic axis coincides with the solar axis of rotation. The resulting displacement-curve was apparently a sine curve. The extension of the theory here given was rendered desirable for the discussion of the observational data relating to the sun's general magnetic field secured by Mr. Hale. I am further indebted to Mr. Anderson for a careful examination and control of the numerous transformations involved in the development of the polarization factor.

SUMMARY

The preceding discussion is concerned with two questions: (*a*) the development of the theoretical curve of displacements produced in the spectrum by the sun's general magnetic field, when observed with the spectrograph and polarizing apparatus of the 150-foot tower telescope; (*b*) the effect upon this curve of the elliptical polarization introduced by the reflections from the silvered surfaces of the coelostat mirrors.

For a normal Zeeman triplet the theoretical displacement curve, equation (31), is a function of the heliographic latitude, the position of the observer, and the solar magnetic elements. It is a sine curve differing but little from $k\Delta = 3 \sin 2\phi$, and has therefore, zero values near the equator and the poles, and maxima (absolute) near 45° N. and S. It is readily adapted to the calculation of the solar magnetic elements.

The effect of the elliptical polarization produced by the tower mirrors is to introduce a factor ϵ, equation (83), which, for normal observing conditions, flattens by a negligible amount the curve of displacements.

MOUNT WILSON SOLAR OBSERVATORY
April 17, 1913

Reviews

Handbuch der Spectroscopie. Von H. Kayser, Professor der Physik an der Universität Bonn. Leipzig: Hirzel, 1912. Band VI. Pp. vi+1067.

The stupendous undertaking upon which Professor Kayser entered more than a quarter of a century ago was the production of a historical compendium which should contain practically all that is known concerning the subject of spectroscopy. The completion of such a work is a noteworthy event in the annals of this science.

The preface of the last volume contains a brief review of the entire project and describes certain changes of plan, all of which is so frank and so full of personal interest that the first three paragraphs are here translated:

When I was writing the preface to the first volume of this work in 1900, I thought the purely physical part of spectroscopy could be treated in four volumes; and I had in mind a fifth volume devoted to spectroscopy as applied in astrophysics. I had hoped to complete the task within ten or twelve years. Already, however, twelve strenuous years have passed and only that portion devoted to physics has been finished. The four volumes originally designed have grown to six. During these twelve years the literature to be worked up has enormously increased; and I may perhaps be allowed to assume that some of this growth in spectroscopic investigation is owing to my work. At that time I stated that I had read and excerpted 7000 spectroscopic papers; but, in the meantime, this number has been doubled. Hence the increase in size and the consequent delay of this work.

As I glance over the astrophysical literature of today it is clearly evident that the single volume which I had intended for this purpose would be quite inadequate; no less than three volumes would suffice, and I should have to devote to it not less than eight years. Nor can I fail to realize that, in the meantime, I have advanced too far in years to undertake such a new work; my university duties have grown, my free time has diminished, my ability to work has waned, and, most important of all, my memory is not to be trusted as formerly: failing which such a work cannot be written. It is not, therefore, without some degree of sadness that I have arrived at the decision to leave these astrophysical applications, in which I have been especially interested, to younger hands, hoping that the proper man will shortly be discovered.

If, in a certain sense, this volume marks the close of my life-work—I took up the subject of spectroscopy in 1880 and since 1887 have devoted to it

almost my entire time—I am nevertheless hoping to pursue these questions still farther. Our scientific output has become so enormous as to render a work of this kind out of date even at the moment of its publication. Time and again I have had to ask the printer to return my manuscript in order that I might include in it some new result which had just appeared and which deserved consideration. In other cases this was impossible. For this reason the volumes published ten years ago are already somewhat antiquated and in need of revision; this I am still hoping to carry out.

Passing now to the text of the volume, this includes the discussion of all elements whose symbols fall within the last half of the alphabet. The first of these is sodium, to which are devoted 127 pages. The treatment of this element—like those of *Rb*, *S*, *Se*, and *Te*—is by Professor Konen of Münster; the number of original papers cited is 678. The behavior of this element is viewed from so many standpoints, such as that of temperature, magnetic field, reversals, character of source, series, fluorescence, dispersive media, etc.; and its history is so completely given, that these 127 pages, by themselves, almost constitute a modern treatise on spectroscopy.

No rigid plan is adopted for the discussion of each element; but the usual procedure is something like the following: the literature of the element is listed in chronological order; next follows the history, general behavior, and peculiarities of the element; this is followed by a description of the line spectrum including tables of wave-lengths as observed in arc and spark; after this, the banded spectra and spectra of compounds. Under these various headings are frequent subdivisions: e.g., five different banded spectra for strontium, eight different types of line spectra for sodium. Nearly all the wave-lengths are necessarily given in terms of Rowland's standards; a few based on the International Unit have appeared in time to be included; such are Kilby's values for titanium, Bachem's for zirconium, and Burns's for iron.

The last two hundred pages, approximately, are devoted to three tables which are certain to be of the utmost value to every spectroscopist who is concerned with either the identification of lines or the determination of wave-lengths.

The first of these tables is a list of iron lines including both secondary and tertiary standards ranging from λ 8863 to λ 2212, distributed at intervals which average about $1\frac{1}{2}$ Å. Intensities are given for both arc and spark. The manner in which the values here tabulated have been obtained may be briefly described by saying that the author has used his best judgment in taking a weighted mean of all the best measures

available. In general the accuracy is estimated at 1 or 2 hundredths of an Ångström unit, and the second decimal is given; the values of the interferometer measures are given to the third decimal, since here the errors are supposed to be less than one-hundredth of an Ångström; some hazy lines are listed only to the first decimal. All lines except a few at the red end have their wave-lengths given both in the International and in the Rowland system. The second table is a list of the "Chief Lines in the Linear Spectra" of the elements, arranged in order of wave-length. Here again wave-lengths are given to the second decimal in both the International and Rowland systems; arc and spark lines are each included on the ground that any distinction between them is artificial—*schädlich und wiedersinnig*. In the last three columns are given the various intensities of each line in the arc, spark, and vacuum tube. One will rarely meet any impurity line not given in this table, and he cannot therefore help wishing that it might, for the sake of convenience, be reprinted as a a separate volume. Here is included the most recent work of Eder and Valenta as well as many yet unpublished measures from the author's own laboratory. The attempt has been to include all lines of intensity of 4 or greater, on a scale of increasing strength running from 1 to 10; but this rule has not been followed in a slavish way. The third and last table includes the wave-lengths of some 3000 edges of banded spectra, giving in addition the source and the direction in which the band shades off, whether toward the red or the violet.

If it were not already true that an easy reading knowledge of German is an absolute necessity for any serious student of spectroscopy, the completion of this handbook would certainly make it true; for this work is the one indispensable treatment of its subject. Professor Kayser surely deserves, and as surely has, the congratulations and the thanks of his fellow-workers and fellow-students in all parts of the world.

HENRY CREW